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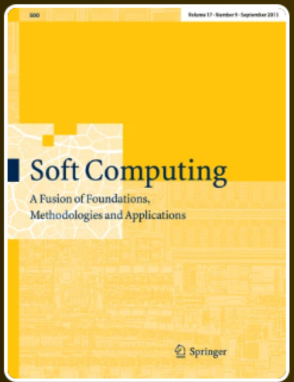

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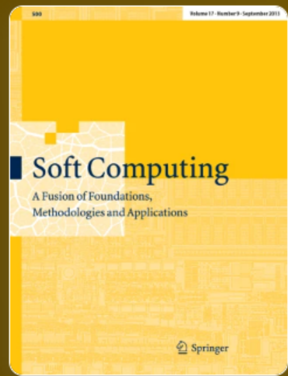

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FUZZY SYSTEMS AND THEIR MATHEMATICS

Selection of cloud service providers using MCDM methodology under intuitionistic fuzzy uncertainty

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Abstract

Cloud computing concept has taken prodigious growth over the last decade. With the vast options of Cloud Service Providers available nowadays and a variety of services and facilities to choose from, it is of paramount necessity to opt for the best cloud service provider based on multiple criteria and requirements ascertained by any organization or an individual. This study selects the cloud service provider based on various conflicting criteria. In this paper, pentagonal intuitionistic fuzzy number (PIFN) with MCDM tool analytic hierarchy process (AHP) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) methods have been used to rank the Cloud Service Providers (CSPs). Firstly, the criteria PIFN weights are calculated using comparison matrices with the help of decision-makers (DMs), and then, FTOPSIS is done to obtain the final ranking. Sensitivity and comparative analyses have been conducted to see the changes in ranking obtained. These analyses help analyze the most sensitive criteria and thus help the researchers mark and evaluate for future scope and further research.

Keywords Cloud Service Providers selection · Intuitionistic fuzzy number · Multi-criterion decision-making method · AHP-TOPSIS method

1 Introduction

We are living in the age of the digital era, where a massive amount of data is available online and offline. The requirement to store, analyze and access the necessary data securely and safely arises every day. We all need a tool that can help any individual or organization to store data online. This modern age requirement had thus necessitated the birth of "Cloud computing," which is basically storing the data on the web cloud, i.e., on shared data servers, and

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provides services like faster access to data, lower operating costs of managing and storing of data, security, and protection of data, etc.

Nowadays, cloud computing technology (CCT) is a well-liked subject matter among researchers and industries. CCT has become a possible preference for businesses and personal services to replace the on-premise IT infrastructure. This technology makes the information technology (IT)-based industries in a remarkable infrastructure and application-oriented services on an online subscription basis ideology. It has changed the understanding of how to acquire computing resources with much adaptability, accessibility, and less organizational effort (Zadeh 1965). These services are fully professional services offered to the subscribers who are interested in running their issues on remote servers when they are not able to execute the problem on local computers anymore. The remote servers are called a cloud. This technological expansion pushes service-given-based companies to acclimatize their services and put forward them to their valuable customers in the cloud. The services through online mode are very popular to the customer and make a huge interest to the online service provider platform. In that context, nowadays, there are many online services offered for the customer but the real challenge is to select the suitable one. Several IT-based companies are changing their business infrastructure and models by giving their products as CCT services. Renowned service providers like AWS, Microsoft, Google and IBM offer similar kinds of services at different levels and prices. The quality and features of such services also differentiate for different companies. The diversity presents a major challenge for clients to determine the most suitable technology that satisfies their main requirements.

Fuzzy set theory and its extension play an important role nowadays. The fuzzy system software has also been developed as the most effective and practical application of the theory and models with fuzzy sets (Chang and Zadeh 1972). These systems can deal with the representation of knowledge and reasoning subject to vagueness and uncertainties for the solution of various kinds of real-life problems. Fuzzy sets and systems are most effective in modelling complex systems and can integrate the human expert system and knowledge with uncertain information. In this regard, fuzzy systems have been implemented and applied in different types of domains of application, among many others, and make suitable decisions in an elegant manner. A fuzzy set consists of the collection of elements where every element has a graded membership value. That means for each and every member, it calculates the likelihood in the set and has degrees of membership values which vary from 0 to 1. The Linguistic variables are also be a representation in the fuzzy set ideology. It may be represented in words rather than numbers (e.g., very low, low, moderate, high, very high, etc.). According to suitable applicability, many studies need to apply the fuzzy logic methodology associated with cloud computing problem to address different types of research issues and challenges with proper solution.

The main objective of this study is to find the solution for selecting the best CCT provider selection (PS) associated with decision-makers fixed alternatives by the help of a proposed decision-making framework under uncertainty.

1.1 Literature review

1.1.1 Fuzzy and Intuitionistic fuzzy set

Uncertainty, indeterminacy, and vagueness are very important terms when associated with the real world conducting tests in terms of observations, measurements and decision-making.

In the classical set theory concept, the membership of elements in a set follows a simple binary logic. That is, either, x. belongs to A., i.e., $x \in A$. or x not belong to A, i.e., $x \notin A$..The idea of corresponding membership grades can be implemented and the concept can be written with the help of characteristic function $\chi_A(x)$.. If $x \in A$., then $\chi_A(x) = 0$ and if $x \notin A$, then $\chi_A(x) = 1$. But in a practical sense, most of the physical phenomena cannot be described by the classical crisp sense since there is some uncertainty. From this point of view, one of the fashionable and efficient logical ideologies to capture the wisdom of uncertainty is "fuzzy set theory," customary by Zadeh(Zadeh 1965). By extending the binary scenarios of "belongs to" and "does not belong to," the fuzzy uncertainty theory provided the notion of the "degree of belongingness." Later, Chang and Zadeh (Chang and Zadeh 1972) constructed a well-structured mathematical version of the fuzzy logic. Then, many researchers (Dubois and Prade 1978; Dubois et al. 2000) contributed much of their effort to enhance the scrupulous domain of information. With the gradual development of the conventional fuzzy philosophy, later different types of fuzzy number ideology exist with different types of applications, such as triangular fuzzy number (Akyar et al. 2012), trapezoidal fuzzy number (Ponnialagan et al. 2018), pentagonal fuzzy number (Chakraborty et al. 2019), dense fuzzy number (Maity et al. 2020a), interval-valued fuzzy number (Mondal 2016), hesitant fuzzy number (Ghorui et al. 2021), lock fuzzy number (Maity et al. 2019), type 2 fuzzy number (Tudu et al. 2021), Hexagonal fuzzy number (Chakraborty et al. 2021), Gaussian fuzzy number (Rahaman et al. 2021), etc.

A major expansion of the fuzzy theory was carried out by the preface of the conception of intuitionistic fuzzy set by Atanassov (Atanassov 1986), which integrated the

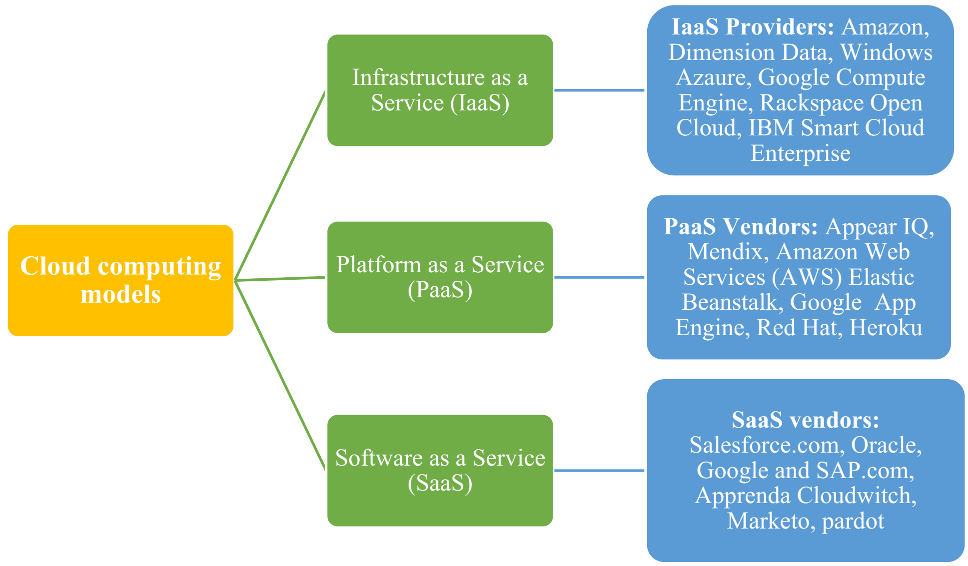

Fig. 1 Model representing the types of Cloud computing services

design to count the "degree of belongingness" and "degree of non-belongingness" as well. For that reason, membership function is considered for degree of belongingness and non-membership function is for non-longingness. There exist different types of intuitionistic fuzzy numbers such as triangular IFS (Mondal and Roy 2014), generalized trapezoidal IFS (Mondal and Roy 2015), cloud-type intuitionistic dense fuzzy number (Maity et al. 2020b), interval-valued IFS (Mondal 2018), pentagonal IFS (Mondal et al. 2018), nonlinear IFS (Mondal et al. 2019), etc.

The application of fuzzy number (Ghorui et al. 2020) and its extension are also very impressive in scientific domain. The author of reference (Xu et al. 2018) applied Delphi method with intuitionistic fuzzy numbers for evaluating the comprehensive product quality for customer satisfaction. The author of reference (Wang et al. 2021) proposed interval 2-tuple linguistic intuitionistic fuzzy numbers and regret theory for scheme selection of design for disassembly (DFD) based on sustainability. The author of reference (Maiti and Roy 2021) used a triangular intuitionistic fuzzy number and proposed defuzzification technique. The introduced ranking method is applied to determine bi-level programming for Stackelberg game. The author of reference (Christi and Kasthuri 2016) and (Pathinathan and Minj 2018) used pentagonal intuitionistic fuzzy numbers, and the latter applied this fuzzy number with Russell's method in transportation problems. The author of reference (Giri et al. 2021) considered nonlinear intuitionistic fuzzy numbers and developed a possibilistic mean by the possibility measure. The implementation of the proposed arithmetic operations is explained by taking a case study of the inventory model. The author of reference (Zhang 2018) used interval-valued intuitionistic fuzzy numbers and introduced Schweizer-Sklar i-norm and Schweizer-Sklar i-conorm. An example was taken to check the validity and effectiveness of the approach. Moreover, we surely say that fuzzy sets and its extensions play an important role for solving several complex problems (Alshammari et al. 2020; Abu Arqub et al. 2021a, 2021b; Abu Arqub 2017; Alzahrani 2021; Ansari et al. 2020; Al-Zahrani 2020; Sahu et al. 2020).

1.2 Concept of cloud computing

Different types of companies nowadays give attention to their core functions by parting Cloud Service Providers (CSPs) to switch their computing belongings. CSPs are vendors who rent to their clients, diverse types of services that are enthusiastically provisioned based on customer's orders in a pay-as-you-go basis strategy.

In terms of applicability and adaptation, cloud computing services are structured (Fig. 1) as follows:

1.3 Infrastructures as a service (IaaS)

IaaS refers to services of cloud computing that offer storage, networking resources, servers, and virtual machines on-demand to the customers. These services are provided over the Internet on a pay-per-use basis. Using IaaS helps the business organization to reduce their expenditure on maintenance of onsite data centers and hardware costs and enables them to increase or decrease resources as per their organizational requirement. IaaS solutions aid the organization in gaining real-time insights into business and bypass the cost and complexity of managing huge data servers.

IaaS provides business solutions to migrate its applications over the cloud, which is scalable and reliable. It is a very useful tool for organizations where the demand for storage is not predictable, and it can increase or decrease the storage or backup and recovery as per the need. IaaS also provides all sorts of the web, networking, and application server support so that the IT team can focus only on the core business solutions of the organization and can easily deploy web apps quickly as and when required without worrying about the infrastructure.

1.4 Platform as a service (PaaS)

PaaS refers to using a platform or environment for software development through a web browser. It includes software development tools, database management systems, middleware, business intelligence services and web development applications. PaaS allows to build, deploy, test, maintain and update web applications. So, the consumer has to manage only the apps and their services and the cloud service provider takes care of the rest of the things.

Developers use PaaS to create applications or cloud-based customized apps which can be created using built-in components of the software. PaaS also helps the analysts in data mining, business predictions, making business decisions based on market insights, data warehousing, etc. Along with all software-related services, it also offers security, storage, workflow, and scheduling.

1.5 Software as a service (SaaS)

SaaS refers to connecting consumers with cloud-based apps using the Internet. It offers facilities such as emails, help desk applications, Microsoft office suite, Customer Relationship Management applications, and online calendars. SaaS is software that is deployed on the server of the Cloud service provider. It enables the consumer to take the app on rent and use it as per his requirement. All the data collected through the app are stored in the data center of the cloud service provider. In turn, the cloud service provider manages the data and also takes care of its security.

Some common form of SaaS which is used by individuals is Outlook, Yahoo Mail, Gmail, Hotmail, etc. This service is provided to individuals free of cost by creating an account with the provider. All the emails that the individual sends or receives are stored in the server of the cloud service provider, which can be accessed anytime, anywhere via the Internet.

Business organizations can use SaaS services to rent apps like ERP or CRM. It needs to subscribe to these apps by paying a subscription amount based on usage. Rest all the factors like software updates, maintenance, and bug fixing are done by the cloud service provider.

1.6 Motivation or objectives of this research

Cloud computing refers to storing data over the Internet or cloud so that it can be accessed anytime, anywhere as per the need if Internet access is available. For MCDM together with cloud computing research any one follow the following Table 1. In today's era of New Normal, all type of business organizations depend largely on the Internet. They are moving their data over the cloud, which gives them more flexibility and data security as well as large storage space without spending huge amounts on infrastructure. Business organizations aim to achieve flexible and scalable platforms with reduced cost and improved quality, and better customer support. The motivation behind this study is to select a good cloud service provider that has a top performance matrix.

There are many competitors in the market of Cloud Service Providers, and each provider comes up with a variety of services and features, which makes them somewhat better than their competitors. Selecting the best provider among so many available providers is an enormous task for any business house. Evaluating the various available options and analyzing their services is the objective of this study. This paper aims at the following:

(1) Selecting different alternatives for Cloud Service Providers and comparing them based on multiple criteria or services provided by them
(2) Selecting different criteria and creating a comparison matrix using PIFN to analyze its result.
(3) Using AHP method to make decisions based on multiple criteria (MCDM Technique).
(4) To compare and analyze which is the best alternative to choose from using numerical applications.

1.7 Novelties of the study

The novelties of the paper are as follows:

(1) Distance measure between two PIFN has been defined and used for the numerical illustration in this research.
(2) Formulae have been developed to determine the pentagonal intuitionistic fuzzy weight of criteria.
(3) Technique has been developed to aggregate the decision-maker's opinions into a single comprehensive significance in terms of PIFN.
(4) The ranking methodology, Fuzzy AHP- TOPSIS and also Fuzzy AHP-COPRAS have been used in this research. Fuzzy AHP-COPRAS is used for the comparative study. A comparative analysis has been conducted with two different fuzzy numbers to understand the difference of PIFN used in this study.

Table 1 Review of MCDM and cloud computing paper

Authors information	The topic of the research	Number of criterion and sub-criterion	Uncertain environment types	MCDM methods used
Büyüközkan et al. (Büyüközkan et al. 2018)	Cloud computing technology selection	6 main criteria and each criterion has 3–6 sub-criteria	Interval-valued Intuitionistic Fuzzy Logic	IVIF AHP, IVIF COPRAS, IVIF MULTIMOORA, IVIF VIKOR
Wibowo et al. Wibowo et al. 2016)	Evaluation of cloud services	5 criteria	Intuitionistic fuzzy numbers	Fuzzy TOPSIS, Choquet integral operator
Jatoth et al. (Jatoth et al. 2019)	Selection of cloud services	7 criteria	Gray Theory	AHP, Gray TOPSIS
Boutkhoum et al. (Boutkhoum et al. 2016)	Selection problem of cloud solution	3 criteria and 10 sub-criteria	Triangular Fuzzy Number (TFN)	Fuzzy AHP, PROMETHEE
Kumar et al. (Kumar et al. 2017)	Cloud service selection	10 criteria	Triangular Fuzzy Number (TFN)	AHP and Fuzzy TOPSIS
Le et al. (Le et al. 2014)	Cloud service selection	9 criteria	Triangular Fuzzy Number (TFN)	ISM and Fuzzy Analytic Network Process (FANP)
Lee & Seo (Lee and Seo 2016)	Cloud service selection	4 criteria	Triangular Fuzzy Number (TFN)	Balanced Scoreboard (BSC) Fuzzy Delphi method, FAHP,
Subramanian & Savarimuthu (Subramanian and Savarimuthu 2016)	Cloud service evaluation and selection in the marketplace	6 criteria and 13 sub-criteria	Triangular Fuzzy Number (TFN)	Fuzzy ANP, Fuzzy TOPSIS, Fuzzy ELECTRE
Ali et al. (Ali et al. 2020)	Measuring the possibility of cloud adoption for software testing	10 criteria and70 sub-criteria	Triangular Fuzzy Number (TFN)	FMCDM
Tanoumand et al. (Tanoumand et al. 2017)	Selecting cloud computing service provider	6 criteria	Triangular Fuzzy Number (TFN)	Fuzzy AHP
Alam et al. (Alam et al. 2018)	Evaluate public cloud computing services	9 main criteria and 30 sub-criteria	Triangular Fuzzy Number (TFN)	FAHP, WASPAS
This paper	Evaluation and selection of CSPs	9 criteria	Pentagonal Intuitionistic Fuzzy Number (PIFN)	FAHP, FTOPSIS

(5) Comparative and sensitivity analyses are conducted to check the robustness and steadiness of the techniques used.

(6) The remainder of the paper is organized in the following way: Sect. 2 briefly describes the concept of fuzzy numbers, PFN, PIFN, and respective arithmetic operations. Distance measure and defuzzification formulae are also represented in this section. It also includes the MCDM technique AHP and Fuzzy TOPSIS. Description of criteria, alternatives, and numerical application are covered in Sect. 3. Sections 4 and 5 represent the sensitivity and comparison analyses, respectively. Section 6 discusses the results and discussion. Finally, the conclusion and future scope are covered in Sect. 7.

2 Pentagonal Intuitionistic Fuzzy Number

Fuzzy set logic was introduced by the author Zadeh (1965) to deal with the impreciseness of real-life situations.

A set $\hat{T}$ is defined as $\hat{T} = \{(\tau, \mu_{\hat{T}}(\tau) : \tau \in \hat{T}, \mu_{\hat{T}}(\tau) \in (0,1)\}$, where $\mu_{\hat{T}}(\tau)$ represents the membership function of $\widehat{T}$, which takes value from zero to one. In real-life situations, where the information is vague and uncertain, fuzzy logic can be efficiently used to deal with these problems.

2.1 Pentagonal Fuzzy Number

(Chakraborty et al. 2019) A pentagonal fuzzy number (PFN) of a fuzzy set is represented as $I = (l_1, m_1, n_1, o_1, p_1); w$, where $l_1, m_1, n_1, o_1, p_1 \in \Re$, the set of real numbers such that $l_1 \le m_1 \le n_1 \le o_1 \le p_1$. The membership of PFN is denoted as

$$U = f(x) = \begin{cases} 0, & \text{for } x \le l_1 \\ w\frac{(x-l_1)}{m_1-l_1}, & \text{for } l_1 \le x \le m_1 \\ w-(1-w)\frac{(x-m_1)}{n_1-m_1}, & \text{for } m_1 \le x \le n_1 \\ 1 & \text{for } x = n_1 \\ 1-(1-w)\frac{(o_1-x)}{o_1-n_1}, & \text{for } n_1 \le x \le o_1 \\ w\frac{(p_1-x)}{p_1-o_1}, & \text{for } o_1 \le x \le p_1 \\ 0 & \text{for } x \ge p_1 \end{cases} \quad (1)$$

Remark 1 If $\mu_{\hat{U}}(x) = 1$, in the closed interval $[m_1, o_1]$, then the pentagonal fuzzy number (PFN) is said to be reduced to the trapezoidal fuzzy number (TrFN).

Remark 2 If $m_1 = n_1 = o_1$, then the PFN is reduced to a triangular fuzzy number (TFN).

Note 1 Fig. 2 denotes PFN, where $l_1 \le m_1 \le n_1 \le o_1 \le p_1$. The variable $'n_1'$ possesses the maximum degree of membership, i.e.,$\mu_{\hat{U}}(x)$ = 1. The variables $'m_1' and 'o_1'$ attain equal membership $\mu_{\hat{U}}(x) = 0.5$, whereas the variables $'l_1'$ and $'p_1'$ have 0 membership value. In other words, membership increases from $'l_1'$, reaches the maximum value at $'n_1'$ and then starts diminishing till $'p_1'$.

2.1.1 Arithmetic operation on PFN

Definition 1 Let us assume two PFN $I = (l_1, m_1, n_1, o_1, p_1)$ and $J = (l_2, m_2, n_2, o_2, p_2)$, then the arithmetic properties between them are defined as follows:

$$1.\ \text{Addition}: (I + J) = (l_1 + l_2, m_1 + m_2, n_1 + n_2, o_1 + o_2, p_1 + p_2) \quad (2)$$

$$\mathbf{2}.\ \text{Subtraction}: (I - J) = (l_1 - p_2, m_1 - o_2, n_1 - n_2, o_1 - m_2, p_1 - l_2) \quad (3)$$

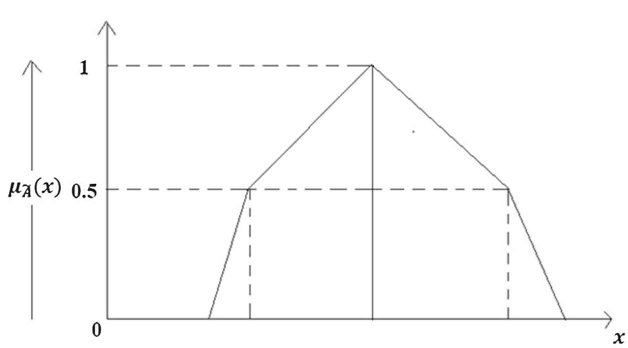

Fig. 2 Graphical representation of PFN

$$\mathbf{3}.\ \text{Multiplication}: (I \times J) = (l_1 l_2, m_1 m_2, n_1 n_2, o_1 o_2, p_1 p_2) \quad (4)$$

$$\mathbf{4}.\ \text{Scalar Multiplication}: aI = (al_1, am_1, an_1, ao_1, ap_1) \quad (5)$$

$$\mathbf{5}.\ \text{Division}: \left(\frac{I}{J}\right) = \left(\frac{l_1}{p_2}, \frac{m_1}{o_2}, \frac{n_1}{n_2}, \frac{o_1}{m_2}, \frac{p_1}{l_2}\right) \quad (6)$$

$$\mathbf{6}.\ \text{Inverse}: I^{-} = \left(\frac{1}{p_1}, \frac{1}{o_1}, \frac{1}{n_1}, \frac{1}{m_1}, \frac{1}{l_1}\right) \quad (7)$$

Tables 2 and 3 describe the linguistic variables in terms of PFN, the latter provides the rating on 1–9 scale. In Table 2, linguistic terms are generally used for rating the alternatives w.r.t. criteria, whereas in Table 3, linguistic ratings are used for the comparison of conflicting criteria.

Note 2 PFNs selected in Tables 2 and 3 are assigned keeping in mind the graphical representation of PFN discussed in Fig. 2. Considering the PFN for the linguistic term 'very high/extremely good,' i.e., $4.3 \le 4.5 \le 4.6 \le 4.8 \le 5$. The maximum degree of belongingness, i.e., $\mu_{\hat{U}}(x)$ = 1 is at point 4.6. The membership function remains at the equal value of 0.5 at the points 4.5 and 4.8, however, considering the variables 4.3 and 5 whose membership value is 0. To summarize, membership value increases from 4.3, attains its pick value at 4.6 and then starts declining to 5. A generalized description of PFN is explained in Note 1.

Note 3 Table 2 denotes the linguistic term used for rating the alternatives w.r.t. the criteria, whereas the linguistic terms used in Table 3 are used for the comparison matrix of criteria.

2.2 Pentagonal Intuitionistic Fuzzy Number (PIFN)

In this section, the definition and pictorial representation (see Fig. 3) of PIFN and its arithmetic operations are defined Nasir and Beenu (2021).

Definition 2 A PIFN $\tilde{I} = \left\{\left(\tilde{l_1}, \widetilde{m_1}, \tilde{n_1}, \tilde{o_1}, \tilde{p_1}\right), w_I, v_I\right\}$ is a distinct fuzzy number on the real set $\Re$. The membership and non-membership functions of PIFN are defined as follows :

Table 2 Linguistic terms expressed in PFN

Linguistic terms	Pentagonal fuzzy number
Very high/extremely good	(4.3, 4.5, 4.6, 4.8, 5)
Moderately high/ very good	(3.7, 3.9, 4, 4.2, 4.3)
Moderate/ good	(2.9, 3, 3.2, 3.4, 3.6)
Low/ poor	(2, 2.2, 2..5, 2.6, 2.8)
Very low/ very poor	(1, 1.5, 1.6, 1.6, 2)

Table 3 Linguistic terms on PFN 1–9 scale

Linguistic terms	1–9 scale	Pentagonal fuzzy number
Equally important (EI)	1	(1,1.5,2,3,5)
Moderately important (MI)	3	(2,4,5,6,6.5)
Strongly important (SI)	5	(3,5,6,7,8)
Very strongly important (VSI)	7	(6,7,7,8,9)
Absolutely important (AI)	9	(7,8,9,9,10)
Moderately not important (MUI)	1/3	(1/6.5,1/6,1/5,1/4,1/2)
Strongly not important (SUI)	1/5	(1/8,1/7,1/6,1/5,1/3)
Very strongly not important (VSUI)	1/7	(1/9,1/8,1/7,1/7,1/6)
Absolutely not important (AUI)	1/9	(1/10,1/9,1/9,1/8,1/7)

$$\mu_I(x) = \begin{cases} 0, & \text{for } x < l_1 \\ W\dfrac{(x-l_1)}{m_1-l_1}, & \text{for } l_1 \le x \le m_1 \\ W + \dfrac{(w_I-0.5)(x-m_1)}{n_1-m_1}, & \text{for } m_1 \le x \le n_1 \\ w_I & \text{for } x = n_1 \\ W + \dfrac{(w_I-W)(o_1-x)}{o_1-n_1}, & \text{for } n_1 \le x \le o_1 \\ W\dfrac{(p_1-x)}{p_1-o_1}, & \text{for } o_1 \le x \le p_1 \\ 0 & \text{for } x > p_1 \end{cases} \tag{8}$$

and

$$\vartheta_I(x) = \begin{cases} 0, & \text{for } x < l_1 \\ 1 - \dfrac{W(x-l_1)}{m_1-l_1}, & \text{for } l_1 \le x \le m_1 \\ \dfrac{W-(W-v_I)(x-m_1)}{n_1-m_1}, & \text{for } m_1 \le x \le n_1 \\ v_I & \text{for } x = n_1 \\ \dfrac{W-(W-v_I)(o_1-x)}{o_1-n_1}, & \text{for } n_1 \le x \le o_1 \\ 1 - \dfrac{W(p_1-x)}{p_1-o_1}, & \text{for } o_1 \le x \le p_1 \\ 0 & \text{for } x > p_1 \end{cases} \tag{9}$$

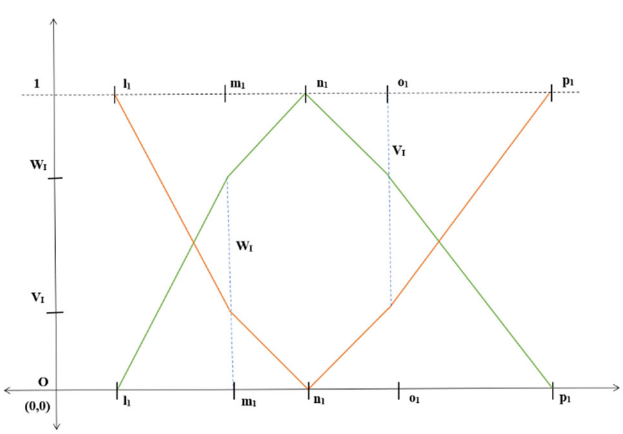

Fig. 3 Graphical representation of PIFN

Throughout the paper, we take the value of $W = 0.5$, where the value of $W \in [0,1]$. The DMs can assume the value depending on the particular research problems.

2.2.1 Arithmetic operations on PIFN

Definition 3 Let us assume two PIFN $A = \left(\widetilde{l_1}, \widetilde{m_1}, \widetilde{n_1}, \widetilde{o_1}, \widetilde{p_1}\right), w_a, v_{a'}$ and $B = \left(\widetilde{l_2}, \widetilde{m_2}, \widetilde{n_2}, \widetilde{o_2}, \widetilde{p_2}\right), w_b, v_{b'}$, then the arithmetic properties between them are defined as follows:

Nasir and Beenu (2021)

$$\text{Addition}: (A+B) = (l_1+l_2,\ m_1+m_2,\ n_1+n_2, o_1+o_2, p_1+p_2), \min\{w_a, w_b\}, \max\{v_{a'}, v_{b'}\} \tag{10}$$

$$\text{Subtraction}: (A-B) = (l_1-p_2,\ m_1-o_2,\ n_1-n_2, o_1-m_2, p_1-l_2), \min\{w_a, w_b\}, \max\{v_{a'}, v_{b'}\} \tag{11}$$

$$\text{Multiplication}: (A \times B) = (l_1 l_2,\ m_1 m_2,\ n_1 n_2, o_1 o_2, p_1 p_2), \min\{w_a, w_b\}, \max\{v_{a'}, v_{b'}\}, A > 0, B > 0 \tag{12}$$

$$\text{Scalar Multiplication: } aI = (al_1,\ am_1,\ an_1, ao_1, ap_1), w_a, v_{a'},\ a \ge 0 \tag{13}$$

$$\text{Inverse}: A^{-} = \left(\frac{1}{p_1}, \frac{1}{o_1}, \frac{1}{n_1}, \frac{1}{m_1}, \frac{1}{l_1}\right), w_a, v_{a'} \tag{14}$$

2.2.2 Distance measure between two PIFN

Distance calculation between two fuzzy numbers is absolutely important in some of the MCDM methods. Distance measure gives an idea of ranking of the alternatives. In this research, TOPSIS approach is used for ranking the CSPs. TOPSIS methodology determines the distance of each alternative from the positive ideal solution (PIS) and negative ideal solution (NIS), respectively. Hence, the final ranking is based on the calculation of relative closeness (RC) which is a formula for distance. As this research incorporates PIFN, the following distance measures are defined to compute the distance between two PIFN.

Definition 4 Let us take two PIFN, $A = \left(\widetilde{l_1}, \widetilde{m_1}, \widetilde{n_1}, \widetilde{o_1}, \widetilde{p_1}\right), w_a, v_{a'}$ and $B = \left(\widetilde{l_2}, \widetilde{m_2}, \widetilde{n_2}, \widetilde{o_2}, \widetilde{p_2}\right), w_b, v_{b'}$, then the distance between two PIFN can be determined by using the following distance methods:

2.2.2.1 Hamming distance

$$d(A,B) = \begin{bmatrix} \left|(1+w_a-v_{a'})\widetilde{l_1} - (1+w_b-v_{b'})\widetilde{l_2}\right| \\ +\left|(1+w_a-v_{a'})\widetilde{m_1} - (1+w_b-v_{b'})\widetilde{m_2}\right| \\ +\left|(1+w_a-v_{a'})\widetilde{n_1} - (1+w_b-v_{b'})\widetilde{n_2}\right| \\ +\left|(1+w_a-v_{a'})\widetilde{o_1} - (1+w_b-v_{b'})\widetilde{o_2}\right| \\ +\left|(1+w_a-v_{a'})\widetilde{p_1} - (1+w_b-v_{b'})\widetilde{p_2}\right| \end{bmatrix} \tag{15}$$

Example 1 Let $a_1 = (1,3,5,7,10); 0.6, 0.2$ and $a_2 = (1,2,3,5,7); 0.7, 0.2$ be two PIFN; then, the distance between them is calculated as follows:

$$d(a_1,a_2) = \frac{1}{5}\begin{bmatrix} |(1+0.6-0.2)1-(1+0.7-0.2)1| \\ +|(1+0.6-0.2)3-(1+0.7-0.2)2| \\ +|(1+0.6-0.2)5-(1+0.7-0.2)3| \\ +|(1+0.6-0.2)7-(1+0.7-0.2)5| \\ +|(1+0.6-0.2)10-(1+0.7-0.2)7| \end{bmatrix}$$

$$= 1.92$$

2.2.3 Reason for considering PIFN for the problem instead of other fuzzy numbers:

Ranking of CSPs under the influence of various conflicting criteria includes hesitancy, uncertainty, and vagueness of the problem. PFNs capture the ambiguity and uncertainty more compared to triangular fuzzy numbers (TFNs) and trapezoidal fuzzy numbers (TRFNs). Incorporation of PFNs with intuitionistic fuzzy numbers, i.e., PIFNs takes the degree of confidence to an enhanced level, as now, the experts or the DMs can express freely not only their degree of assurance but also the degree of non- assurance. In real-life situations, when an expert or group of experts have been given the platform to express their sureness and hesitancy, that indicates rational thinking, which leads to optimal decision-making. Thus, in this paper, PIFN has been integrated with MCDM methods AHP, TOPSIS, and COPRAS for obtaining the weight of the criteria and ranking of the CSPs.

2.3 PIFN- AHP method

AHP (analytic hierarchy process) is a famous mathematical tool which was first developed by the author of Wind and Saaty (1980), a scientific method used in multi-criteria decision-making (MCDM). This method helps decision-makers to resolve intricate problems with heuristic methods. Calculations of criteria's weights are the key points for ranking risk factors. AHP constructs the problem hierarchy with the creation of comparison matrices to give subjective findings about the factors which are considered vastly responsible for ranking. In this paper, FAHP is (see Fig. 4) used rather than AHP as Fuzzy takes under advisement the ambiguity and impreciseness of the decision experts. The steps for FAHP are described below.

Step 1 Construction of a comparison matrix in terms of PIFN by a decision expert or a group of decision experts using Table 3. The linguistic ratings assigned by DMs are transformed into PIFN. The aggregation of the opinions is done by using Eq. 22.

Step 2 Defuzzification of PIFN.

Defuzzification or score value of PIFN can be calculated using the following formula:

$$S_p = \frac{l+m+3n+o+p}{6}(1+w_a-v_{a'}) \tag{16}$$

Step 3 Normalization of each of the defuzzified matrix

$$N_{gh} = \frac{n_{gh}}{\sum_{g=1}^{m} n_{gh}}, \text{ where } g = 1,2,\ldots,m; g = 1,2,\ldots,n \tag{17}$$

Step 4 Estimation of criteria priority weight (P.W)

$$P.W = \frac{p^{th} rootvalue}{\sum p^{th} root} \tag{18}$$

Step 5 Determination of the consistency index $(C.I)$ of the matrix

$$(C.I) = \frac{\delta_{max} - n}{n-1}$$

where $\boldsymbol{n}$ denotes the size of the matrix considered in the study.

Step 6 Calculation of consistency ratio (C.R)

$$(C.R) = \frac{C.I}{R.I} \tag{19}$$

where random index (R.I) value changes with the size of the matrix "$\boldsymbol{n}$."

The value of C.R ≤ 0.1 denotes the matrix to be consistent.

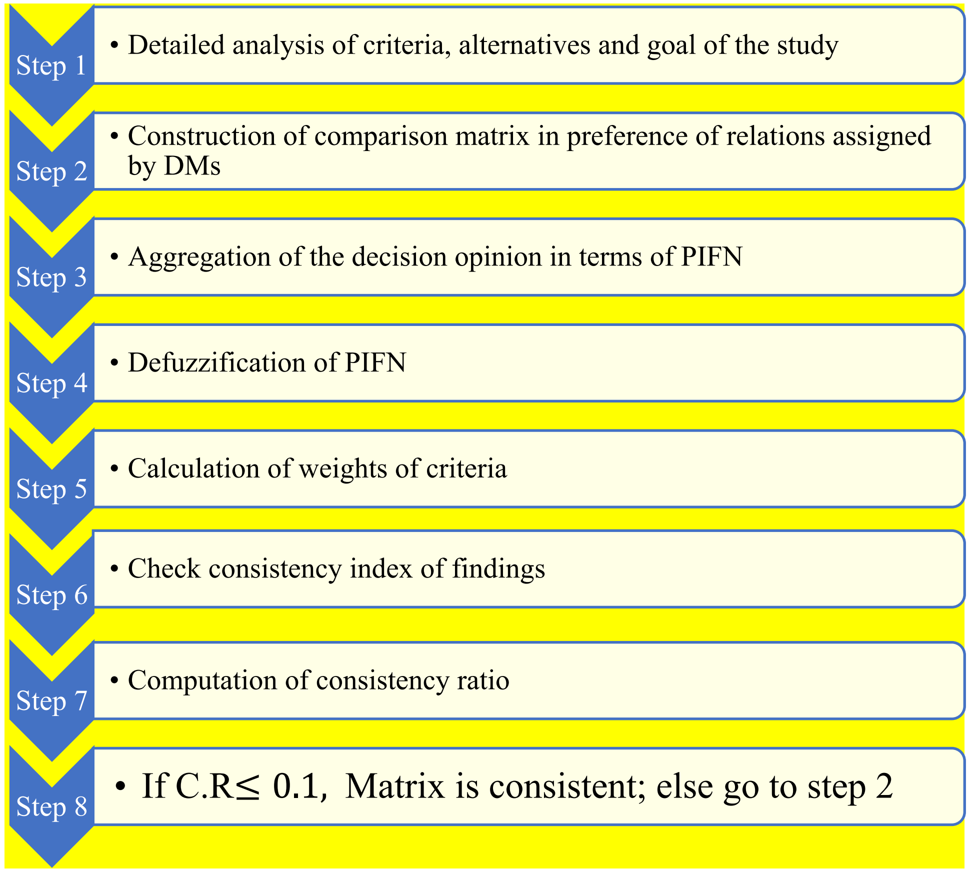

Fig. 4 Schematic structure of AHP for strategy

2.4 Determination of pentagonal intuitionistic fuzzy weights of criteria

Step 1 The geometric mean value of PIFN is obtained using

$$M_g = \left(\prod_{h=1}^{n} y_{gh}\right); \min_{h=1,2,..,n} w_{gh}, \max_{h=1,2,..,n} v_{gh}, g = 1,2,\ldots,m \quad (20)$$

Step 2 Summation of each M_g using Eq. 7.

Step 3 Calculation of the inverse of each M_g using Eq. 11 and thereby arranged in increasing order.

*Step 4*To determine the pentagonal intuitionistic fuzzy weight of criteria using the following equation:

$$W_g = M_g * (M_1 + M_2 + \cdots + M_i)^{-1} \quad (21)$$

2.5 PIFN- TOPSIS approach

The TOPSIS MCDM technique is one of the extensively used methodologies, introduced by Hwang and Yoon (Hwang and Yoon 1981) to rank the alternatives, and thus, DMs have a clear idea about the most preferred alternative. The TOPSIS process is categorized as a distance measurement method in which the ideal alternative obtained is nearby to the positive ideal solution (PIS) and extreme from the negative ideal solution (NIS). The linguistic terms assigned by the DMs can be captured better with fuzzy TOPSIS (FTOPSIS). The fuzzy approach is useful in dealing with the hesitancy and uncertainty of the DMs and the conflicting criteria. In this research, for the ranking of CSPs, which is dependent on multiple conflicting criteria, the MCDM method fuzzy TOPSIS is used. The PIFN has been incorporated with the classical TOPSIS technique to obtain FTOPSIS (see Fig. 5) formulae. The fuzzy logic extends our goal to obtain more sensitive results in this regard.

For the ranking of Cloud Service Providers (CSPs), the MCDM technique TOPSIS introduced by the authors of Hwang and Yoon (1981) is applied. The PIFN-TOPSIS methodology is described in the following steps:

Step 1 Determination of alternatives and their preferential linguistic ratings in terms of PIFN.

Let us assume a set of 'm' CSPs $p_1, p_2, \ldots., p_m$ and 'n' criteria $f_1, f_2, \ldots., f_n$. Let $D_1, D_2, \ldots., D_k$ be the number of decision-makers (DMs). Thus, the DMs assign PIFNs as their decision for the alternatives depending on different criteria.

Step 2 Aggregation of decisions of DM assigned in PIFN using the following equation:

$$\begin{cases} \tilde{l}_{gh} = \min_{k=1,2,\ldots,K} \tilde{l}_{hik} \\ \tilde{m}_{gh} = \min_{k=1,2,\ldots,K} \tilde{m}_{hik} \\ \tilde{n}_{gh} = \sqrt[K]{\prod_{k=1}^{K} \tilde{n}_{hik}} \\ \tilde{o}_{gh} = \max_{k=1,2,\ldots,K} \tilde{o}_{hik} \\ \tilde{p}_{gh} = \max_{k=1,2,\ldots,k} \tilde{p}\, hik \\ w_{gh} = \min_{k=1,2,\ldots,k} w_{hik} \\ v_{gh} = \max_{k=1,2,\ldots,k} v_{hik} \end{cases} \tag{22}$$

Step 3 Normalization of the PIFN, using the formula:

$$\breve{N} = [n_{gh}]_{mn}, g = 1,2,\ldots,m; h = 1,2,\ldots..,n$$

$$N^{B}_{gh} = \left(\frac{\tilde{l}_{gh}}{\tilde{p}^*}, \frac{\tilde{m}_{gh}}{\tilde{p}^*}, \frac{\tilde{n}_{gh}}{\tilde{p}^*}, \frac{\tilde{o}_{gh}}{\tilde{p}^*}, \frac{\tilde{p}_{gh}}{\tilde{p}^*}\right); w_{hi}, v_{hi} d \in B.C, \tilde{p}^* = \max p_{gh}$$

$$N^{NB}_{gh} = \left(\frac{\tilde{l}^*_h}{\tilde{p}_{gh}}, \frac{\tilde{l}^*_h}{\tilde{o}_{gh}}, \frac{\tilde{l}^*_h}{\tilde{n}_{gh}}, \frac{\tilde{l}^*_h}{\tilde{m}_{gh}}, \frac{\tilde{l}^*_h}{\tilde{l}_{gh}}\right); w_{hi}, v_{hi} d \in N.B.C, \tilde{l}^*_h = \min \tilde{l}_{gh} \tag{23}$$

where B.C and N.B.C signify the benefit criteria and non-benefit criteria, respectively.

Step 4 To calculate the weighted fuzzy normalized matrix, the criteria's fuzzy weightage is multiplied by the normalized fuzzy value. For the product of two PIFN, Eq. 12 needs to be used.

$$W = [WN_{gh}]_{mn} g = 1,2,\ldots,m; h = 1,2,\ldots..,n$$

where

$$FW_{gh} = \breve{N}_{gh} \times \widehat{W_h}, \quad g = 1,2,\ldots..,m; h = 1,2,\ldots,n \tag{24}$$

Step 5 Computation of the fuzzy positive ideal solution (FPIS) (P^+) and fuzzy negative ideal solution (FNIS)(N^-), where t^+_g denotes the maximum value of t_{gh} and t^-_g denotes the minimum value of t_{gh}:

$$P^+ = \left(a^+_1; \min_g w_{g1}, \max_g v_{g1}\right), \left(a^+_2; \min_g w_{g2}, \max_g v_{g2}\right), \ldots, \left(a^+_n; \min_g w_{mn}, \max_g v_{mn}\right) = \left\{\begin{pmatrix} \max t_{gh} | h \in M_{BC}), \\ (\min t_{gh} | h \in M_{NBC} \end{pmatrix}\right\},$$

$$N^- = \left(a^-_1; \min_g w_{g1}, \max_g v_{g1}\right), \left(a^-_2; \min_g w_{g2}, \max_g v_{g2}\right), \ldots, \left(a^-_n; \min_g w_{mn}, \max_g v_{mn}\right) = \left\{\begin{pmatrix} \min t_{gh} | h \in M_{BC}), \\ (\max t_{gh} | h \in M_{NBC} \end{pmatrix}\right\} \tag{25}$$

where M_B denotes the benefit criteria, and M_{NB} denotes the non-benefit criteria.

Step 6 Determination of the distance measure of all alternatives from the PIS and NIS. The two Hamming distances for an individual alternative can be computed as follows:

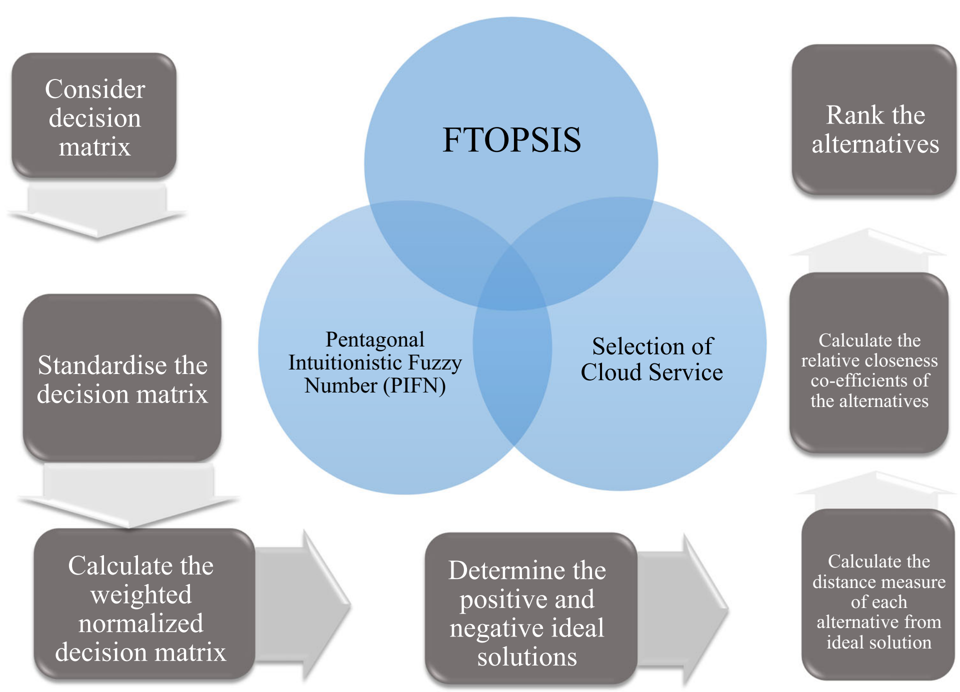

Fig. 5 A schematic structure of FTOPSIS strategy

$$\breve{M}_g^+ = \sum_{h=1}^{n} d(t_{gh}, t_g^+),\ g = 1, 2, \ldots..., m$$

$$\breve{M}_g^- = \sum_{h=1}^{n} d(t_{gh}, t_g^-),\ g = 1, 2, \ldots..., m \tag{26}$$

where (.,.) denotes the Hamming distance between two fuzzy numbers. Using Eq. 15, the distance between two PIFN can be computed.

Step 7 Determination of the relative closeness to the ideal alternatives:

$$RC_g = \frac{\breve{M}_g^-}{\breve{M}_g^- + \breve{M}_g^+},\ g = 1, 2, \ldots., m \tag{27}$$

Step 8 Ranking of the alternatives:

The alternatives are ranked based on the value determined by RC_g.The larger value of RC_g represents the better alternatives.

3 Pentagonal Intuitionistic Fuzzy MCDM methodology for the ranking of CSPs

3.1 Pseudocode depicting the empirical study application

The research model under consideration involving 'g' number of CSPs based on 'h' number of criteria is represented below. The input taken in our study is the preferential linguistic terms assigned by DMs. These variables are converted to PIFN for obtaining the output, i.e., the ranking of the treatment options.

g = Cloud Service Providers.

h = Number of criteria.

g*h = Size of the matrix.

Input The preferential rating matrix in terms of PIFN.

Output The ranking order of the CSPs in the TOPSIS approach.

1. **for**(g = 1 to m, h = 1 to n)**do.**
2. Generate PIFN by DMs.

For every given criterion, create a matrix and compare the given criteria with each other using linguistic terms on PIFN 1–9 scale.

3. Calculating criteria weight in PIFN by using FAHP.
4. Use PIFN AHP methodology to check whether the matrix is consistent or not.
5. If the matrix is consistent, calculate PIFN-TOPSIS for ranking of CSPs.

Else, Go back to step 4.

6. Construct normalized values NZ_{ef}

$overset\_N = [n_{gh}]_{mn}$ $, g = 1, 2, \ldots, m; h = 1, 2, \ldots..., n;$

$$N_{gh}^{B} = \left(\frac{\tilde{l}_{gh}}{\tilde{p}^*}, \frac{\tilde{m}_{gh}}{\tilde{p}^*}, \frac{\tilde{n}_{gh}}{\tilde{p}^*}, \frac{\tilde{o}_{gh}}{\tilde{p}^*}, \frac{\tilde{p}_{gh}}{\tilde{p}^*}\right); w_{hi}, v_{hi} d \in B.C, \tilde{p}^* = \max p_{gh},$$

$$N_{gh}^{NB} = \left(\frac{\tilde{l}_h^*}{\tilde{p}_{gh}}, \frac{\tilde{l}_h^*}{\tilde{o}_{gh}}, \frac{\tilde{l}_h^*}{\tilde{n}_{gh}}, \frac{\tilde{l}_h^*}{\tilde{m}_{gh}}, \frac{\tilde{l}_h^*}{\tilde{l}_{gh}}\right); w_{hi}, v_{hi} d \in N.B.C, \tilde{l}_h^* = \min \tilde{l}_{gh};$$

7. Generate weighted normalized value $FW_{gh} = \breve{N}_{gh} \times \hat{W}_h;$
8. Calculate $(FPIS)$ and $(FPIS)$

$$P^+ = \left(a_1^+; \min_g w_{g1}, \max_g v_{g1}\right), \left(a_2^+; \min_g w_{g2}, \max_g v_{g2}\right), \ldots., \left(a_n^+; \min_g w_{mn}, \max_g v_{mn}\right) = \left\{\left(\max t_{gh} | h \in M_{BC}\right), \left(\min t_{gh} | h \in M_{NBC}\right)\right\},$$

$$N^- = \left\langle \left(a_1^-; \min_g w_{g1}, \max_g v_{g1}\right), \left(a_2^-; \min_g w_{g2}, \max_g v_{g2}\right), \ldots, \left(a_n^-; \min_g w_{mn}, \max_g v_{mn}\right) \right\rangle \left\{\left(\min t_{gh} | h \in M_{BC}\right), \left(\max t_{gh} | h \in M_{NBC}\right)\right\};$$

9. Calculate the distance measure of each CSPS from $(FPIS)$ and $(FPIS)$

$$\breve{M}_g^+ = \sum_{h=1}^{n} d(t_{gh}, t_g^+),$$

$$\breve{M}_g^- = \sum_{h=1}^{n} d(t_{gh}, t_g^-),$$

10. Compute relative closeness $RC_g = \frac{\breve{M}_g^-}{\breve{M}_g^- + \breve{M}_g^+};$
11. **end for.**

Note 3.1 The decision-makers are the people who are using cloud services in either the organization where they work or in their business establishment 3.2 Flowchart of the application taken in the proposed research. The methodology are taken for solving the addressed issues are graphically shown in the Fig. 6 as follows:

3.2 Description of the cloud service providers (CSPs) taken in this study

Cloud Space is networked storage, or disk space, available over a specific network—the Internet.

Cloud Service Providers are organizations who rent out the cloud space to other organizations or individuals and provide them with cloud-based platforms where data can be stored or accessed. The requirement of CSP arises when a business unit wants to reduce the cost of hardware, servers and storage space and transfer all the data online so that it can be accessed from anywhere and anytime.

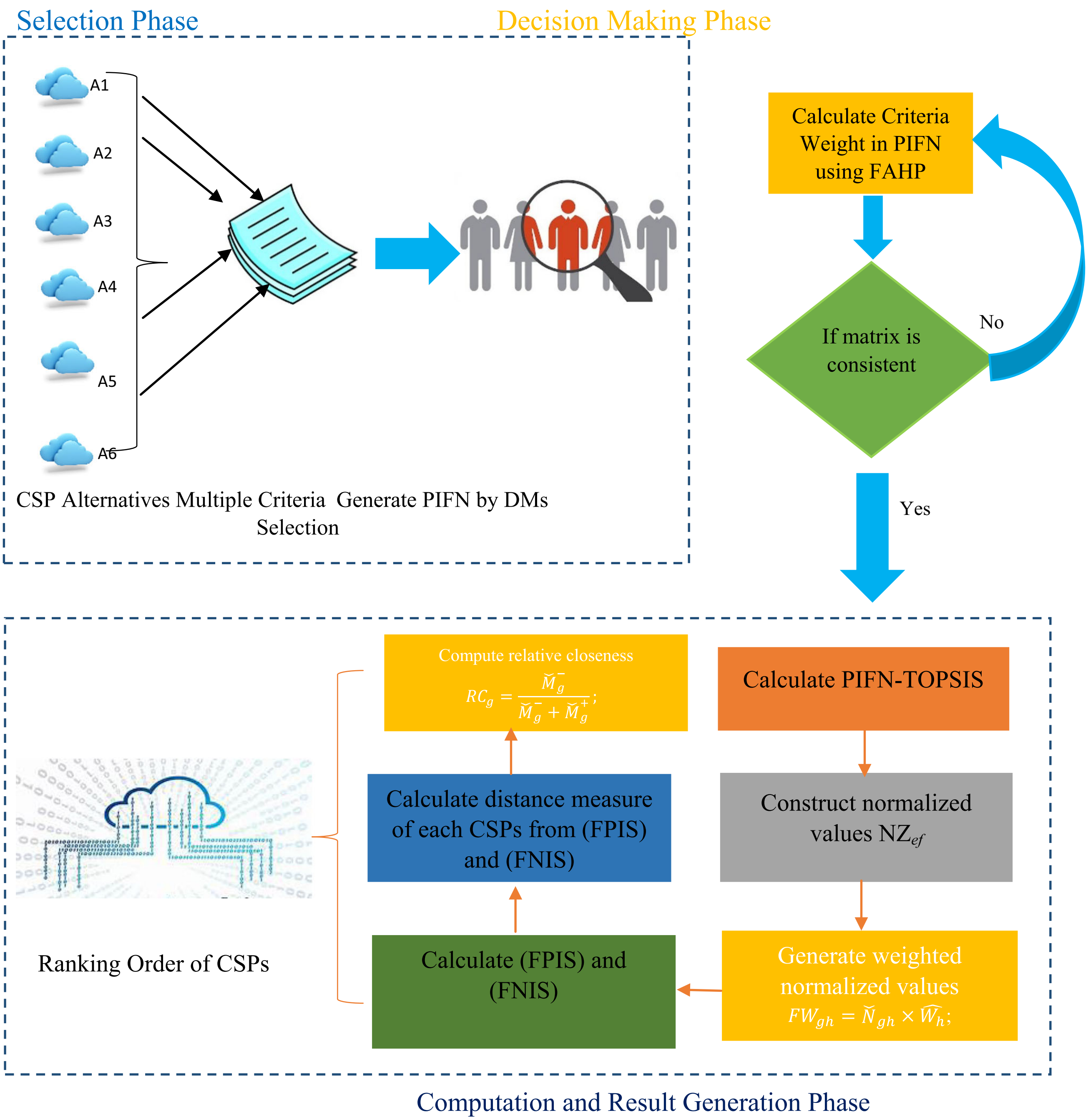

Fig. 6 Flowchart depicting the methodology used in the study

Transferring data to the cloud gives the organization or an individual an option to reduce the storage cost, streamline work, and collaborate with other members remotely.

CSP helps in managing all the information technology-related activities like backup, security, support, and maintenance of data being saved in cloud space. These companies help in establishing and managing public and private clouds or on-demand computing services which can be customized as per the requirement of the customer. Cloud Service Technologies are required to ascertain the best suitable services for the consumers. It empowers individuals or organizations to develop computing solutions quickly at a reasonable cost. Among the pool of CSPs available which provides services globally, we are selecting six top alternatives for Cloud Service Providers and will compare them on various parameters. In this study, the alternatives selected are some renowned CSPs, the names have not been mentioned as they may have a problem ranking them without any authorization.

3.3 Cloud service provider selection criteria

Cloud Service Provider selection criteria are taken into consideration after a detailed evaluation by a group of decision expert. All the criteria have been taken keeping in mind the general requirement of cloud users by doing a survey with few decision-makers and industry colleagues. We also did secondary research in order to validate our findings through case paper studies and Internet research. In this segment, we have briefly explained the criteria taken in this study.

1. Cloud Security/Privacy—Protection of data against data loss, theft or leakage. Cloud security consists of policies, procedures and technologies that work to protect the data and customers' privacy.

   Data privacy and data security are considered to be the top priority for any organization. CSP also needs to abide by the industry and regulatory requirements so that any kind of data breach can be avoided. So, any organization needs to clearly understand the security measures offered by the CSP and also the mechanism that is being used to protect the organization's data and applications. In case the organization deals in sensitive data like finance or national security, then it is imperative to have data privacy and data security at the highest level.
2. Pricing—Cost of cloud services and discounts if any are based on business models and their framework. While it may not be a big factor for any small usage organization, it plays a significant role if cloud usage is significant or a big organization.

   One needs to look into the pricing structure and its various components charged by CSP. Some CSPs determine the price on an hourly usage basis, some charges on a minutes basis or some charges on a pay-as-per-go basis. So, the pricing component is one of the important factors for any customer.
3. Downtime—It is the total time period in which services are not available due to unexpected circumstances such as outages, maintenance activities or maintenance or updates of the providers. Many natural disasters also lead to the downtime of cloud solutions.

   The unavailability of services being provided by the CSP may increase the cost to the client. It may affect their normal working routines and they might incur heavy losses due to it. It also impacts the reputation of the provider, as well as the customer.
4. Support Services—These are the activities which provide QoS and high-performing networks and also ensure a high-quality end-user experience. Support services are important in the deployment of CSP in any organization. One has to check whether CSP's cloud architecture and support services offered are suitable for the organization's workload and roadmap. Customization or any recoding is required by CSP to fit into the organization's requirements. Additionally, one has to enquire about the extra or any hidden charges and response time for any additional support services or not?
5. Portability—It is the ability to be easily deployed or migrated to a new user or location without any integration issues. It ensures smooth change from one environment to another with minimum disruption. (Büyüközkan et al. 2018).

   There may be chances that the data of customers might be lost due to moving the data from one platform to another. To avoid this, CSP should provide an environment or an API (Application Program Interface) which helps to retrieve the data from the old platform/source to a new platform (Cloud Standards Customer Council 2017).
6. Scalability—It is the ability to increase or decrease resources or services to meet the changing demands. In cloud computing, scalability refers to the ability to increase workload with existing hardware resources. This is important to evaluate in order to determine whether a system can handle a large number of application requests. (Jeong 2013; Almishal and Youssef 2014).
7. Disaster recovery—Any organization going for CSP implementation needs to understand the disaster recovery and data recovery options and CSPs ability to support data preservations. Does CSP have any backup plan, data restoration plan, integrity checks, and the additional cost associated with data recovery, if any?
8. Deployment and upgrades—For deployment, there are mainly three basic variants that can be distinguished as Public Cloud (offering of services from a freely accessible provider), Private Cloud (services which can only be accessed by one company) and Hybrid Cloud (i.e., getting best from the former two and combining it). Also, one needs to see whether CSP is giving regular updates or not and are they included or charged separately.
9. Service Level Agreements (SLAs) —It is the agreement between the provider and customer about the quality and reliability of service. It describes all the negotiations and terms and conditions of the contract (Ashraf 2014). SLAs could be very complex as there are no standards available in the cloud industry on how to construct SLAs and define it. Anyone take the different alternative as different service providers and give inputs with respect to the alternatives. Using the proposed methodology any one can rank the alternative

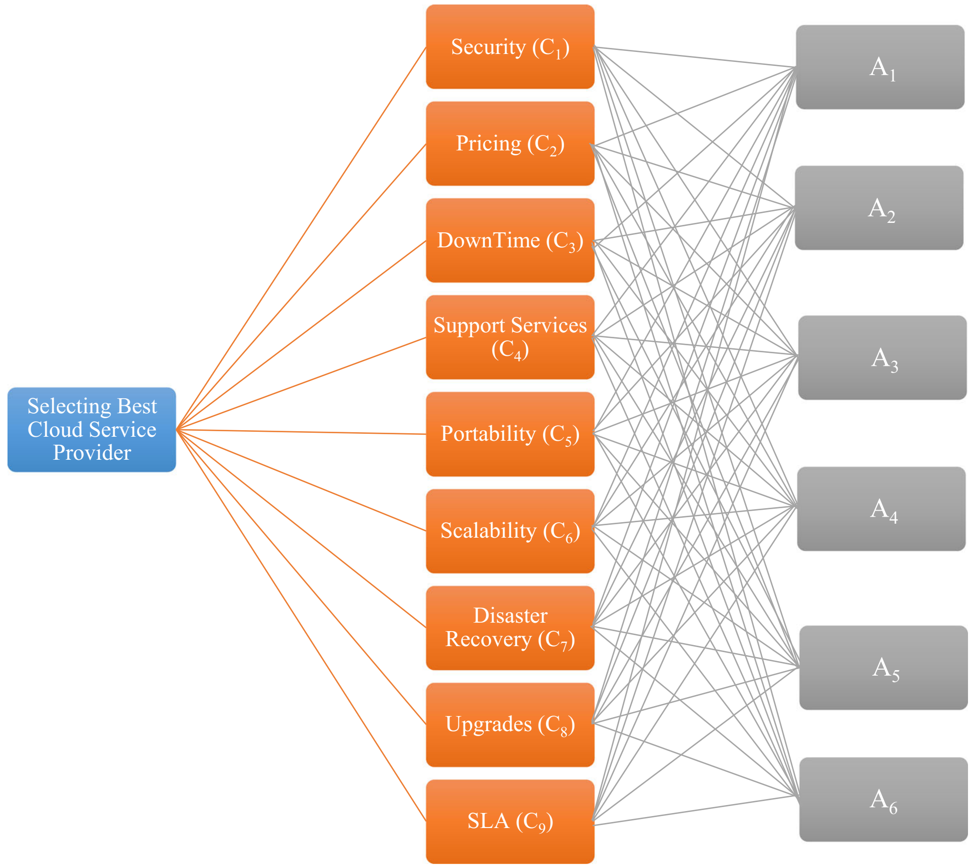

Fig. 7 Hierarchical framework representing the criteria and alternatives taken in this research

Table 4 Linguistic variables rating in terms of PIFN

Linguistic terms	1–9 Scale	Pentagonal intuitionistic fuzzy number
Equally important (EI)	1	$\langle(1,1,1,1,1); w_a, v_{a'}\rangle$
Moderately important (MI)	3	$\langle(2,4,5,6,6.5); w_a, v_{a'}\rangle$
Strongly important (SI)	5	$\langle(3,5,6,7,8); w_a, v_{a'}\rangle$
Very strongly important (VSI)	7	$\langle(6,7,7,8,9); w_a, v_{a'}\rangle$
Absolutely important (AI)	9	$\langle(7,8,9,9,10); w_a, v_{a'}\rangle$
Moderately not important (MUI)	1/3	$\langle(1/6.5, 1/6, 1/5, 1/4, 1/2), w_a, v_{a'}\rangle$
Strongly not important (SUI)	1/5	$\langle(1/8, 1/7, 1/6, 1/5, 1/3)w_a, v_{a'}\rangle$
Very strongly not important (VSUI)	1/7	
Absolutely not important (AUI)	1/9	$\langle(1/10, 1/9, 1/9, 1/8, 1/7)w_a, v_{a'}\rangle$

easily. In the Fig. 7 hierarchical framework of the criteria and alternatives taken in the paper are shown.

Note 4.1 In the criteria given above, pricing and downtime are non-benefit criteria, while security/privacy, support services, portability, scalability, upgrades, disaster recovery, and SLA are benefit criteria.

Table 4 illustrates the PIFN linguistic terms taken in this study for the calculation of criteria's weight. Since the membership degree and non-membership value may vary for each DMs, the generalized value $(w_a, v_{a'})$ is represented in Table 4.

Note 3 The transpose of the matrix is represented in Table 5.

The linguistic data collected by the DMs in Table 5 are transformed to PIFN. The individual PIFN assigned by the DMS is integrated into a single PIFN by using Eq. 22. The PIFN weights are calculated using the steps described in sub-Sect. 3.4 of this paper. Table 6 depicts the final PIFN weights which will be used further for ranking the alternatives using the FTOPSIS approach.

Table 7 represents the weights of the criteria calculated by using the MCDM tool FAHP. For the calculation of criteria's crisp weights, firstly, the PIFN is defuzzified

Table 5 Criteria to criteria comparison were conducted by three DMs

Criteria	Decision-makers (DMs)	Security (c^1)	Pricing (c^2)	Downtime (c^3)	Support Services (c^4)	Portability (c^5)	Scalability (c^6)	Disaster Recovery (c^7)	Upgrades (c^8)	SLA (c^9)
Security (c^1)	DM1	EI	1/AI	EI	1/AI	EI	1/AI	EI	1/AI	1/AI
	DM2	EI	1/AI	1/AI	1/AI	1/AI	1/AI	EI	1/AI	1/AI
	DM3	EI	1/AI	1/AI	1/AI	1/AI	EI	EI	EI	1/AI
Pricing (c^2)	DM1	AI	EI	EI	VSI	VSI	1/SI	AI	VSI	EI
	DM2	AI	EI	VSI	VSI	SI	1/SI	AI	VSI	VSI
	DM3	AI	EI	VSI	SI	VSI	VSI	AI	AI	VSI
Down Time (c^3)	DM1	AI	EI	EI	EI	EI	1/VSI	AI	EI	EI
	DM2	AI	1/VSI	EI	1/VSI	VSI	1/VSI	EI	EI	EI
	DM3	AI	1/VSI	EI	SI	EI	VSI	AI	AI	EI
Support Services (c^4)	DM1	AI	EI	EI	EI	EI	1/VSI	AI	EI	EI
	DM2	AI	1/VSI	VSI	EI	EI	EI	AI	VSI	EI
	DM3	AI	1/SI	EI	EI	EI	VSI	AI	AI	EI
Portability (c^5)	DM1	AI	1/VSI	EI	EI	EI	1/VSI	EI	EI	1/VSI
	DM2	AI	1/SI	EI	EI	EI	1/SI	AI	VSI	1/VSI
	DM3	AI	1/VSI	EI	EI	EI	AI	AI	AI	EI
Scalability (c^6)	DM1	AI	SI	VSI	VSI	VSI	EI	AI	VSI	VSI
	DM2	AI	SI	VSI	EI	SI	EI	AI	VSI	SI
	DM3	AI	1/VSI	1/VSI	1/VSI	1/AI	EI	EI	EI	VSI
Disaster Recovery (c^7)	DM1	EI	1/AI	1/AI	1/AI	EI	I/AI	EI	1/AI	1/AI
	DM2	EI	1/AI	EI	1/AI	1/AI	1/AI	EI	1/AI	1/AI
	DM3	EI	1/AI	1/AI	1/AI	1/AI	EI	EI	EI	1/AI
Upgrades (c^8)	DM1	AI	1/VSI	VSI	EI	EI	1/VSI	AI	EI	EI
	DM2	AI	1/VSI	EI	1/VSI	1/VSI	1/VSI	AI	EI	EI
	DM3	EI	1/AI	1/AI	1/AI	1/AI	EI	EI	EI	1/AI
SLA (c^9)	DM1	AI	EI	EI	EI	VSI	1/VSI	AI	EI	EI
	DM2	AI	1/VSI	EI	EI	VSI	1/SI	AI	EI	EI
	DM3	AI	1/VSI	EI	EI	EI	1/VSI	AI	AI	EI

Table 6 The PIFN weights of criteria taken in this research

Criteria	PIFN weights of Criteria
Security (c^1)	$\langle(0.182, 0.217, 0.376, 0.574, 0.677); 0.5, 0.4\rangle$
Pricing (c^2)	$\langle(0.005, 0.006, 0.017, 0.046, 0.059); 0.5, 0.4\rangle$
Down time (c^3)	$\langle(0.009, 0.010, 0.042, 0.165, 0.189); 0.5, 0.4\rangle$
Support services (c^4)	$\langle(0.013, 0.016, 0.044, 0.129, 0.151); 0.5, 0.4\rangle$
Portability (c^5)	$\langle(0.016, 0.019, 0.063, 0.208, 0.242); 0.5, 0.4\rangle$
Scalability (c^6)	$\langle(0.006, 0.007, 0.030, 0.176, 0.214); 0.5, 0.4\rangle$
Disaster recovery (c^7)	$\langle(0.077, 0.086, 0.261, 0.560, 0.662); 0.5, 0.4\rangle$
Upgrades (c^8)	$\langle(0.029, 0.033, 0.114, 0.444, 0.518); 0.5, 0.4\rangle$
SLA (c^9)	$\langle(0.016, 0.020, 0.047, 0.104, 0.120); 0.5, 0.4\rangle$

using Eq. 16, and then, the AHP tool is applied to get the priority weights. The AHP technique is discussed in sub-Sect. 3.3 of this research paper.

Table 7 and Fig. 8 represent the crispified weights of criteria calculated by using FAHP.

Note 3 The scores and chart represent the criteria 'security (c^1)' with the highest score followed by 'disaster recovery (c^7),' 'upgrades (c^8),' 'portability (c^5),' 'downtime (c^3),' 'scalability (c^6),' 'support services (c^4),' 'SLA)' and 'pricing (c^2).'

According to the score value shown in above Table 5, it is evident that 'Security' is the most vital attribute for selecting a Cloud service provider with the crisp weight of '0.294.' Protection of data against data loss, theft or

Table 7 Criteria weights were obtained by using FAHP

Security (c^1)	Pricing (c^2)	Down time (c^3)	Support services (c^4)	Portability (c^5)	Scalability (c^6)	Disaster recovery (c^7)	Upgrades (c^8)	SLA (c^9)
0.294	0.013	0.055	0.049	0.073	0.053	0.278	0.141	0.043

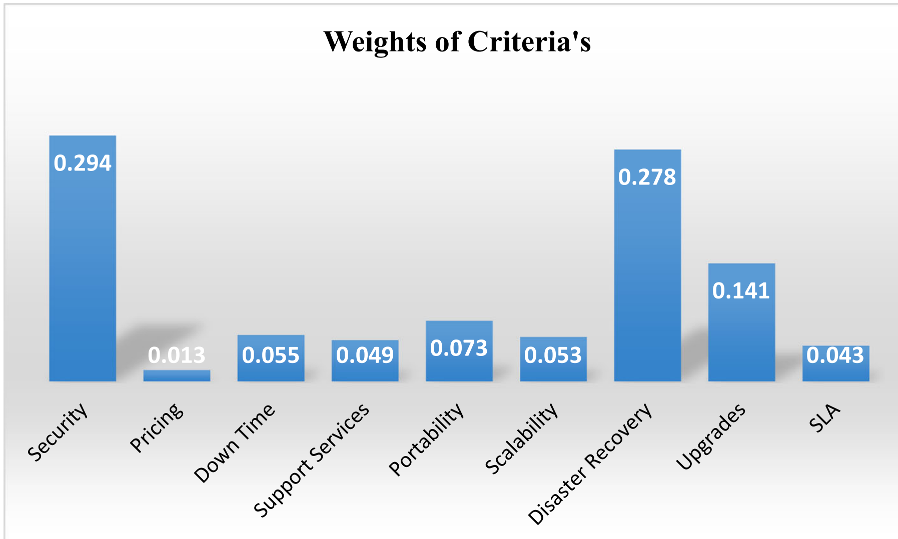

Fig. 8 Clustered column chart depicting the weights of criteria obtained using FAHP

Table 8 Linguistic variables rating in terms of PIFN for alternative rating w.r.t criteria

Linguistic terms	Pentagonal intuitionistic fuzzy number
Extremely good (EG)/ extremely high (EH)	$\langle(5, 5.4, 5.5, 5.6, 6); w_a, v_{a'}\rangle$
Very good (VG)very high (VH)	$\langle(4.2, 4.5, 4.7, 4.9, 5); w_a, v_{a'}\rangle$
Good (G)/ high (H)	$\langle(3, 3.3, 3.6, 3.8, 4); w_a, v_{a'}\rangle$
Poor (P)/ low (L)	$\langle(2.2, 2.3, 2.5, 2.9, 3); w_a, v_{a'}\rangle$
Very poor (VP)/ very low (VL)	$\langle(1, 1.2, 1.5, 1.7, 2); w_a, v_{a'}\rangle$

Table 9 Alternatives linguistic ratings assigned by DMs w.r.t criteria

Alternatives	A_1		A_2		A_3		A_4		A_5		A_6	
Decision-makers (DMs)	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2
Security	EG	EG	EG	EG	EG	EG	EG	EG	VG	VG	VG	VG
Pricing	L	H	H	H	H	H	H	H	L	H	L	L
Down time	VL	VL	L	L	L	L	H	L	L	L	H	H
Support Services	EG	EG	EG	EG	EG	EG	VG	VG	VG	VG	EG	VG
Portability	EG	EG	VG	VG	EG	EG	EG	EG	VG	VG	VG	VG
Scalability	EG	EG	G	G	VG	VG	EG	VG	VG	VG	VG	VG
Disaster Recovery	EG	EG	EG	EG	VG	VG	G	VG	G	VG	VG	G
Upgrades	EG	EG	EG	EG	EG	EG	VG	VG	VG	VG	G	G
SLA	EG	EG	EG	EG	EG	EG	EG	EG	VG	VG	G	G

leakage is a topmost priority so that any kind of data breach can be avoided. This is followed by Disaster recovery with the weight of '0.278.' For any organization; data recovery options, data preservation and a backup plan for data restoration, and integrity checks are crucial. Next comes upgrades which are also important and need to be done in regular intervals so that it complies with the ongoing market trends along with the dynamism of the macro-

Table 10 Representation of distance measure from PIS and NIS, relative closeness and ranks of the alternatives

Alternative	D +	D-	Relative closeness	Rank
A_1	0.059	0.293	0.833	1
A_2	0.063	0.288	0.820	2
A_3	0.080	0.272	0.773	3
A_4	0.123	0.228	0.649	4
A_5	0.232	0.119	0.339	5
A_6	0.269	0.083	0.235	6

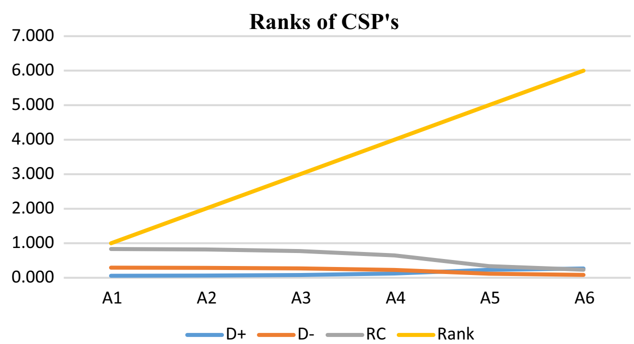

Fig. 9 Line chart representing the distance measure, relative closeness and rankings of the CSPs

Table 11 Rankings obtained under sensitivity analysis

Alternatives	Case 1	Case 2	Case 3
A_1	2	4	1
A_2	1	1	2
A_3	3	2	3
A_4	4	3	4
A_5	5	6	5
A_6	6	5	6

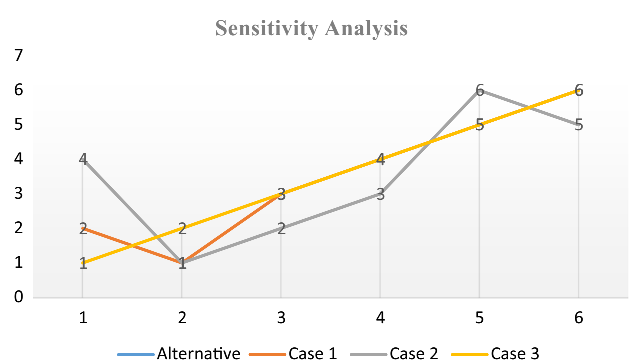

Fig. 10 Line chart illustration of rankings obtained under sensitivity analysis

environment. We use the linguistic variable rating in terms of PIFN for the alternative with respect to criterion in Table 8. DMs rating for alternative are mention in Table 9.

3.4 TOPSIS for final ranking of the CSPs (Numerical study)

Step 1. Formation of integrated PIFN using Eq. 22.

Step 2 Calculation of normalized matrix using equation 23.

Step 3 Computation of weighted normalized matrix using equation 24.

Note 4 The weighted normalized matrix is constructed using the product of PIFN weights of criteria obtained and the normalized matrix.

Step 4 PIS and NIS are calculated using equation 25.

Step 5 Distance measures of each CSPs are calculated from the PIS and NIS using Eq. 15.

Step 6 Lastly, the relative closeness is determined for each alternative using Eq. 27 as mention in Table 10. The higher value of *RC* represents the optimal CSPs (see the Table 10 and Fig. 9).

4 Sensitivity Analysis

Sensitivity analysis is carried out by interchanging the PIFN weights of the most sensitive criterion. Three different cases have been undertaken, and thus, the rankings obtained under these cases are represented in Table 11. Graphically, these rankings are illustrated with the help of a line chart in Fig. 10.

Case 1 Interchange of PIFN weights of support services and pricing—On interchanging the given values by decision-makers for support services and pricing, we found that the ranking of Alternative 1 has changed, while the rest others are not showing any change. This shows that pricing is one of the factors which is responsible to change the selection of cloud service providers A1 and A2.

Table 12 Ranks of the alternatives using FCOPRAS and FTOPSIS

Alternatives	Rank (FCOPRAS)	Rank (FTOPSIS)
A_1	1	1
A_2	2	2
A_3	3	3
A_4	4	4
	5	5
	6	6

Case 2 Interchange of PIFN weights of upgrades and Downtime—This case shows a drastic change in the ranking of all the alternatives except A2. Minimal downtime and regular updates should be the main criteria of cloud service provider but according to the decision-makers, these two criteria have different values if we consider different CSP alternatives. As, downtime is considered a vital factor for selecting CSP, interchanging its value with other criteria shows a substantial change.

Case 3 Interchange of PIFN weights of portability and scalability—As both the criteria portability and scalability are at par, the changes in the ranking of alternatives are very negligible.

The ranking obtained under the first two cases depicts that the alternative A_1 ranking order is changed from the first position to 2nd and 4th place in the two cases, respectively. This change in ranking under the change of two sensitive criterion represents the importance of these criteria. Considering the case 3, it is easy to understand from Table 11 and Fig. 10 that the same ranking is obtained as the methodology used in this paper. The interchange of weight of criteria's portability and scalability shows no alteration in ranking of the alternatives. This indicates the equivalency of these two criteria. Thus, these three cases in sensitivity study deliver an overall thought to the CSPs about the significance of individual criteria. Moreover, in future, CSPs can efficiently utilize this idea to obtain maximum benefit.

Remark *The three cases are considered as per the opinion of the decision expert. The most sensitive criteria are taken into account, and accordingly, their weights are interchanged to obtain the ranking. This mechanism is utilized to analyze the change in the pattern of ranking and to get a clear and deeper concept of the most important criteria.*

5 Comparative Analysis

The comparative study has been divided into two segments. In the first analysis, our methodology, i.e., FTOPSIS ranking, has been compared with the FCOPRAS MCDM ranking tool. In the second segment, the fuzzy number, i.e., PIFN, which has been applied in this study, has been compared with the trapezoidal intuitionistic fuzzy number (TRIFN), triangular intuitionistic fuzzy number (TIFN). Thus, the ranking obtained under these two different concepts has been discussed in the following subsections.

5.1 Comparative study with FCOPRAS technique

The complex proportional assessment (COPRAS) methodology was first developed by the authors of reference (Zavadskas et al. 1994). Fuzzy COPRAS is a comprehensive method of COPRAS technique, widely applicable to decision-making problems. It is a stepwise ranking and evaluation technique for the alternatives with reference to significance and utility degree. Table 12, Figs. 11, 12 and 13 demonstrate the ranking obtained using the existing MCDM tool FTOPSIS with FCOPRAS. The latter approach has been used for comparing with the existing one, and the observation or the ranking obtained is the same.

5.2 Comparative study with different fuzzy numbers

This section depicts Table 13 and Fig. 12, which are obtained under the ranking of CSPs under trapezoidal intuitionistic fuzzy numbers (TrIFN), (Parvathi and Malathi 2012) and (Rezvani 2013) and triangular intuitionistic fuzzy numbers (TIFN), (Wang et al. 2013) and (Li 2010) and the ranking is compared with PIFN which is used in this research. The author of reference (Vashishtha and Susan 2022) developed multi-lexicons adaptive neuro fuzzy inference system (MultiLexANIFS), which integrates inputs from different lexicons to carry out the sentiment analysis of social media posts. The existing method, i.e., by use of PIFN, the alternative A_1 ranks the highest, followed by $A_2 > A_3 > A_4 > A_5 > A_6$. The rank obtained under TrIFN shows that the rank of the alternative $A_1\&A_2$ remains the consistent position of 1st and 2nd followed by $A_3 > A_4 > A_6 > A_5$. In TIFN, the ranks of the alternatives are interchanged, A_2 being the first followed by $A_1 > A_3 > A_4 > A_6 > A_5$.

Remark *The ranking obtained under comparative analysis in this section depicts same ranking when FTOPSIS is compared with FCOPRAS. This indicates the equivalency of the two methods. Thus FTOPSIS technique is used for ranking and FCOPRAS technique isfor comparative study. In numerical we get consistent results for both techniques. The comparative analysiswas also done with respect to different types of intuitionistic fuzzy numbers i.e., TrIFN and TIFN insubsection 5.2* $A_1, A_2, A_3 and A_4$ *but interchange of ranking for the* $A_5 and A_6$ *alternatives. In comparison of PIFN with TFN, it is seen that the alternative* A_2 *scored first rank followed by* A_1 *scoring second rank. The other alternatives ranking obtained under TFN hold consistent rank with TrFN. The ranking obtained under PIFN with MCDM tool TOPSIS can be said as the best technique because PIFN incorporates the degree of hesitancy and vagueness of the DMs in an optimal way compared to other type of Intuitionistic fuzzy number.*

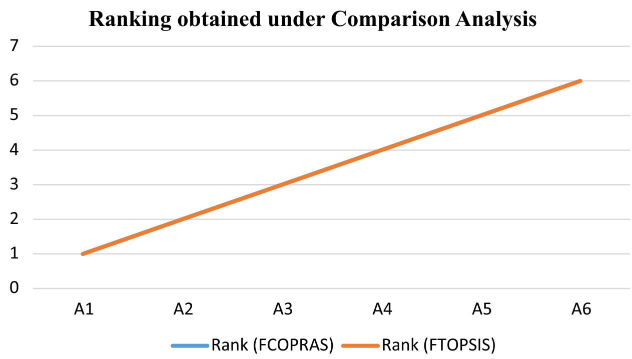

Fig. 11 Line chart representation of rankings obtained under comparative analysis

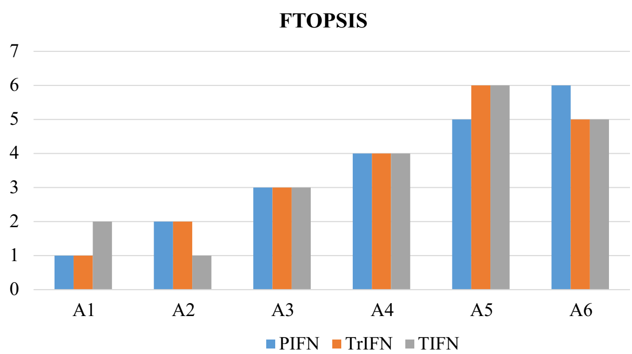

Fig. 12 Bar graph representation of rankings obtained under the comparative analysis of different types of intuitionistic fuzzy number

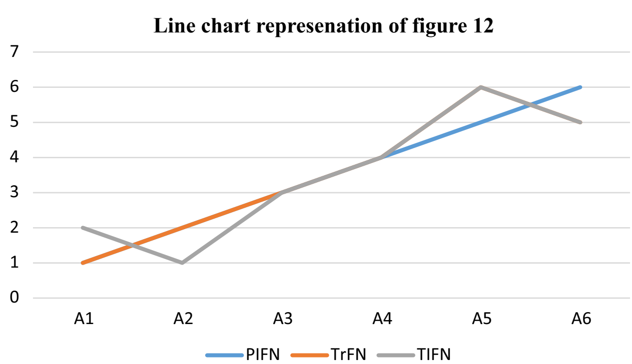

Fig. 13 Line chart representation of rankings obtained under the comparative analysis of different types of intuitionistic fuzzy number

6 Main findings

This paper reveals the rankings of the Cloud Service Provider alternative using fuzzy AHP and fuzzy TOPSIS methods. The ranking technique FTOPSIS is further compared with the other MCDM technique FCOPRAS. The comparative study is presented in Sect. 5. The comparative study has been integrated into two sections. The first sub-Sect. 5.1 depicts the comparison of MCDM FTOPSIS used in this paper with FCOPRAS. The ranking obtained under these two MCDM shows that the alternatives A_1 and A_2 obtain the rankings 1 and 2, respectively. Thus, the FTOPSIS technique for ranking used in this study and the FCOPRAS for comparison yields consistent rankings of the CSPs. In the second segment, the intuitionistic fuzzy number, i.e., TrIFN and TIFN.

Table 13 Ranks of the alternatives using different types of intuitionistic fuzzy numbers

Alternatives	PIFN	TrIFN	TIFN
A_1	1	1	2
A_2	2	2	1
A_3	3	3	3
A_4	4	4	4
A_5	5	6	6
A_6	6	5	5

Using TOPSIS, the final ranking has been done, which shows that Alternatives 1 and 2, i.e., are the best Cloud Service Providers taking into consideration all the factors responsible for selecting the best alternative. The sensitivity analysis shown in Sect. 4 shows that in case 1 and 3, where the most sensitive criteria's fuzzy weights are interchanged, also alternative 1 and 2 obtain consistent ranks of 1 and 2, which implies that they are the top giants of the market due to the fact that it provides excellent services along with customer satisfaction and any business house can consider it reliable to store and access their important data.

7 Conclusion and future research scope

This paper primarily focuses on the use of intuitionistic fuzzy numbers with fuzzy AHP-fuzzy TOPSIS tool for obtaining the weights of criteria and ranking of CSPs. Formulae are developed using PIFN for determine the weight of criteria and also aggregate the decision-maker's estimation into a single comprehensive value. Further, Fuzzy AHP- TOPSIS and Fuzzy AHP- COPRAS have been used to determine the final ranking of the alternatives. The requirement to move the business to the cloud has increased manifold due to the fact that it reduces the operational cost and provides flexibility to the business to grow its functional areas. So, the opinions of different decision-makers have been considered to determine the best alternative. Although the views of decision-makers are highly influenced by their working environment and the size and severity of business, the attempt to determine the alternative ranking is done. Comparative and sensitivity analyses have been conducted to check the robustness and steadiness of the techniques used.

In the future study, the researchers can use different types of fuzzy numbers depending on the problem and availability of the data. The different number of criteria and alternatives can be added or eliminated. Diverse MCDM tools such as WASPAS, ELECTRE, PROMETHEE, VIKOR methodology can be used in future.

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Data availability All data supporting the results reported in the article are present in the paper.

Declarations

Conflict of interest The authors declare that there is no conflict of interest in this study.

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IMPACT FACTOR 3.8

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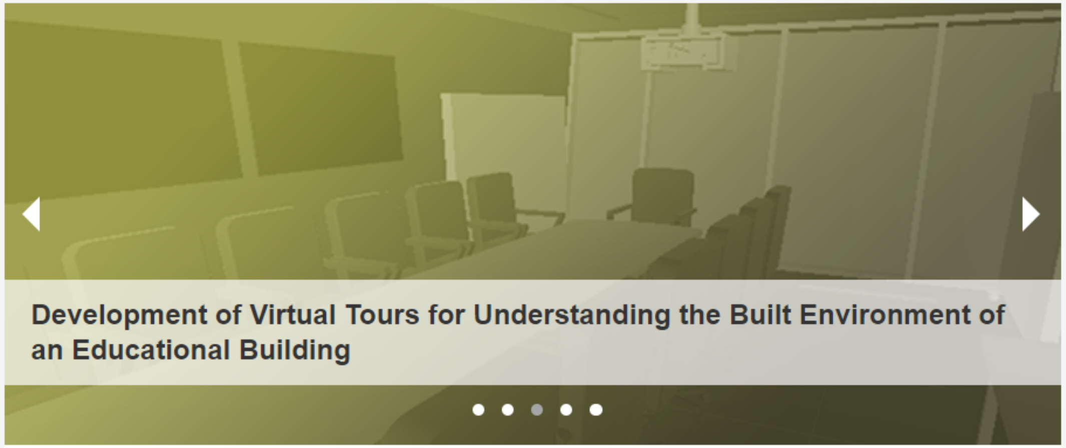

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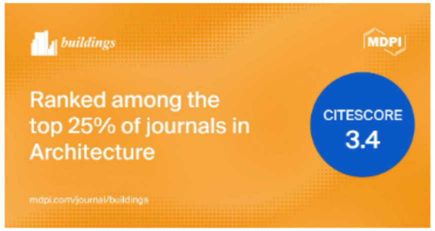

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Article

Optimal Site Selection for Women University Using Neutrosophic Multi-Criteria Decision Making Approach

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Abstract: Site selection for an institute or a university is a challenging task. The selection of sites for setting up a new university depends on multiple criteria. In backward, under privileged area people's perception towards the co-educational universities and women universities are different. Poor families with their conservative mentality possess inhibitions while sending their girl child to co-educational universities as they have concerns about safety, security and family honor. Hence many attributes which are not so important for co-educational universities are more pertinent for women university. In this research paper, we have considered a model for selecting women's university sites in different backward locations in the state of West Bengal, India. This model incorporated different types of uncertainty related to site selection. Ten important criteria are chosen for the selection of sites. To capture the uncertainty of the problem, trapezoidal neutrosophic numbers are used along with the Multi-criteria Decision Making tool Analytic Hierarchy Process (AHP) for obtaining criteria weights. Finally, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and COmplex PRoportional ASsessment (COPRAS) are applied for ranking of the sites. Comparative and sensitivity analyses are conducted to check the steadiness of the techniques used.

Keywords: neutrosophic number; TRNNs; AHP; TOPSIS; COPRAS; university site selection

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1. Introduction

The site selection problem is quite significant in today's world. Location selection for setting up an industry, real estate, hospitality management, or other cases that require proper data, decision experts, future perspective, establishment cost, etc. Moreover, there exist several criteria which make the Decision Maker (DM) select the optimal alternative. Ranking the sites requires a mathematical understanding of the problem. In this context, multi-criteria decision-making (MCDM) can play an important role. Optimal selection or ranking of various disparate sites in decision-making is complicated as it depends on multiple conflicting criteria. Obtaining of criterion's weight is a major part of the DMs. The first step in decision-making is to integrate the opinion of decision experts in linguistic rating. The linguistic rating may not always be transformed to a fixed scale, as the decision experts may consider uncertainty, hesitancy, and vagueness. In this context, the researchers need to solve the MCDM techniques for the site selection problem in an uncertain, hesitant environment.

1.1. Motivation and Novelties of the Study

- MCDM tools in the neutrosophic environment have been applied in different areas, but the literature survey reflects that minimum work has been done on women's university site selection problem (any other type of university site selection also). So, this study focuses on this problem and aims to fill the gap in the literature. It provides encouraging results.

- New De-Neutrosophication technique for trapezoidal neutrosophic number is developed and further used in this study model.
- Finding the trapezoidal neutrosophic numbers (TrNNs), weights are constructed in a new way and applied in the proposed application model.
- A model for Women University site selection is proposed by taking important criteria, and ranking of the sites is conducted using an uncertain MCDM method.
- Questionnaires were made regarding different locations and their attributes. Various experts were questioned, such as transportation engineers, architects, environmental engineers, civil engineers, geologists, environmental experts, and municipal officials. Their opinion about the criteria that correspond to different locations has been taken into consideration as the input and has been aggregated to solve the problem.
- AHP technique has been used to obtain the crisp weight of the criteria, and for consistency check, two MCDM tools, namely Neutrosophic TOPSIS and Neutrosophic COPRAS have been applied for ranking of the sites.
- Sensitivity analysis has been calculated to measure the change in ranking and to check the robustness and steadiness of the methods used.

1.2. Structure of the Paper

In this research, Section 1 covers the introduction and the literature review of neutrosophic number and its application in MCDM as described in Section 2. Section 3 covers preliminaries and different operations of neutrosophic numbers with distance and de-neutrosophication technique of TrNN. MCDM mathematical formulation is mentioned in Section 4. In particular, NAHP is described in Section 4.1, TrNN weight is expressed in Section 4.2, Section 4.3 described NTOPSIS and NCOPRAS described in Section 4.4. The model for setup and Criteria of the application are shown in Sections 5 and 6, respectively. Data collection and Numerical calculations are mentioned in Sections 7 and 8, respectively. Sensitivity analysis is described in Section 9. Finally, the conclusion and future research scope are discussed in Section 10.

2. Literature Review

In this section, we shall review some related published work and some key terms used in this paper.

2.1. Multi Criterion Decision Making

The application of MCDM tool in the fuzzy environment and its importance in real-life situations are discussed in this section. This section depicts the theoretical development and the work done related to these techniques in the last few years in Table 1.

Table 1. Literature review with respect to different MCDM techniques and applications.

	Authors	Years	MCDM Methods	Application Area
[1]	Serrai, W. et al.	2017	AHP, ANP, SAW, TOPSIS, VIKOR, PROMETHEE, MAUT, ELECTRE, BWM & COPRAS	Web service selection
[2]	Jayant, A. et al.	2018	ELECTRE, PROMETHEE, VIKOR, TOPSIS	Business competitive environment
[3]	Zain, Z.M.	2018	Fuzzy TOPSIS	Evaluation of the quality of online information on breast cancer
[4]	Chattopadhyay, A. et al.	2018	Fuzzy TOPSIS	Supplier selection
[5]	Mitra, S. et al.	2018	TOPSIS	Best domestic Refrigerator selection
[6]	Sa L.K. et al.	2018	TOPSIS	Green Material Selection
[7]	Balioti, V. et al	2018	TOPSIS	Spillway Selection
[8]	Khan, S.A. et al.	2018	TOPSIS	Supply Chain Management

Table 1. *Cont.*

	Authors	Years	MCDM Methods	Application Area
[9]	Masum, A.K.M. et al.	2019	AHP-TOPSIS	Ranking Human Capital
[10]	Min Oo, H. et al.	2019	Fuzzy AHP, TOPSIS	Destination selection
[11]	Gholap, A.B. et al	2019	AHP & TOPSIS	Ranking Condition monitoring
[12]	Negi, N. et al	2019	AHP, ARAS, TOPSIS & VIKOR	Quality of Service(QoS) based Web services
[13]	Singh, M. et al	2019	TOPSIS, MTOPSIS, FTOPSIS	Raw material selection in pulp and paper making industry
[14]	Song, Y. et al	2019	TOPSIS	Financial risk prediction
[15]	Cheng, C. et al	2020	MCDM	Selecting a supplier
[16]	Youssef, A.E.	2020	BWM, TOPSIS, AHP	Cloud Service Selection
[17]	Zulqarnain, R.M. et al	2020	TOPSIS	Selection of a car
[18]	Abdelli, A. et al	2020	TOPSIS	Web services characterized by Quality of service (QoS)
[19]	Lin, M. et al	2020	TODIM	Evaluating IoT Platforms
[20]	Pangaribuan, I. et al	2020	SAW, TOPSIS, WPM	Auction application
[21]	Raju, S.S. et al	2020	AHP, TOPSIS, MOORA	Ranking of Al-CSA
[22]	Dr. Kashid, U. et al	2021	AHP, TOPSIS	Players performance evaluation and selection in IPL
[23]	Lee, W.H. et al	2021	TOPSIS, WSM	Sustainable building materials supplier selection
[24]	Trung, DO D.	2021	EDAS, MARCOS, TOPSIS, MOORA & PIV	The weights of surface roughness calculation and finding MRR
[25]	Vassoney, E. et al	2021	SAW, WPM, AHP, TOPSIS, VIKOR, ELECTRE III & SHARE MCA	The optimal flow of downstream of a small run of the river HP plant
[26]	Lata, S. et al	2021	Fuzzy TOPSIS	Selection of Machine Tool
[27]	Basaran, S. et al	2022	Fuzzy TOPSIS	Mobile Mathematics Learning Application Selection
[28]	Ukpanyang, D. et al	2022	PROMETHEE	Waste-to-Energy Technologies for Slum/Informal Settlements selection
[29]	Olgun, M. et al	2022	TOPSIS	Multicriteria Group Decision Making
[30]	Boix-Cots, D. et al	2022	MIVES	Different construction problem (Review paper)
[31]	Casanovas-Rubio, M. M. et al	2019	MIVES	Sustainability assessment of trenches
[32]	Pujadas, P. et al	2018	MIVES	Urban-pavement conditions evaluation
[33]	Pons, O. et al	2016	AHP & MIVES	Sustainability of green building
[34]	Pujadasa, P. et al	2017	MIVES	Evaluation, prioritization and selection of public investment projects
This paper		2023	NAHP, NTOPSIS & NCOPRAS	Women's university site selection

2.2. Neutrosophic Set

The concept of belongingness, non-belongingness, and indeterminacy of Neutrosophic Set was first developed by F. Smarandache [35]. In this study, the Degree of truth, degree of falsity and indeterminacy of every element in the set are considered.

2.3. Neutrosophic Set with MCDM Techniques

The MCDM method is quite a popular approach when it comes to real life complexities. A few real-life problems are also associated with uncertain data. Due to that reason, the uncertain MCDM methodology is developed. After the invention of Neutrosophic set theory, this approach has becomes very successful. The following review is based on the theoretical development and applications in Neutrosophic MCDM in the last five years. Please refer Table 2.

Table 2. Literature review based on theoretical developments by neutrosophic MCDM techniques along with application area.

	Authors	Years	Neutrosophic Number/Set Used	MCDM Technique Used	Application Area
[36]	Zhang et al.	2014	Interval neutrosophic sets	INN Aggregation Operators based MCDM	Money investing problem
[37]	Ren et al.	2017	Single Valued Neutrosophic Set (SVNS)	Prioritized Weighted Geometric (SVNPWG) Operator based MCDM	Selection of an investment company
[38]	Garg & Nancy	2018	Single Valued Neutrosophic Number (SVNN)	Prioritized Muirhead Mean based MCDM	Finding an appropriate Information Technology (IT) software company
[39]	Sodenkamp et al.	2018	Single Valued Neutrosophic Set (SVNS) and Group Decision Making Aggregation	NS-based GMCDM approach	Unequal voting powers be respon-sible for the assessment of ranking alternatives
[40]	Nabeeh et al.	2019	Triangular Neutrosophic Numbers	AHP	Selection of IoT based Enterprises
[41]	Wang et al.	2019	Interval Neutrosophic Sets	Improved cosine similarity measure based MCDM method	Supplier selection
[42]	Garg & Nancy	2019	Single Valued Neutrosophic Set (SVNS), Divergence measure developed	Divergence measure based TOPSIS method	Finding an appropriate Information Technology (IT) software company
[43]	Zeng et al.	2020	Single Valued Neutrosophic Set (SVNS)	Correlation based TOPSIS	Finding an appropriate Information Technology (IT) software company
[44]	Jiao et al.	2020	Interval Neutrosophic Number (INN)	Induced Generalized Interval Neutrosophic Choquet Integral based MCDM	Country selection for investment
[45]	Duong & Thao	2021	Entropy based Neutrosophic Numbers	TOPSIS	Market segment selection and evaluation
[46]	Ye et al.	2021	Neutrosophic enthalpy set	Aggregation operator and score function based MCDM method using the algebraic and the Einstein t-norms and t-conorms	Car selection
[47]	Hezam et al.	2021	Generalized triangular neutrosophic number	Neutrosophic AHP-TOPSIS	Prioritized peoples group selection for vaccine
[48]	Jafar et al.	2021	Neutrosophic Hypersoft Sets	Similarity measures based MCDM	Renewable energy source selection
[49]	Rani et al.	2021	Single Valued Neutrosophic Set (SVNS)	SWARA & CoCoSo	Renewal Energy Source Selection
[50]	Abdullah et al.	2021	Single Valued Neutrosophic Set (SVNS)	DEMATEL	Identification of Influential Criteria in Sub- Contractors Selection
[51]	Elhosini et al.	2021	Single Valued Neutrosophic Set (SVNS)	TOPSIS, PROMETHEE	Selection of Wind Energy Power Plant Location
[52]	Ridvan et al.	2021	Single Valued Neutrosophic Set (SVNS)	Divergence, Projection, Likelihood (DPL)- TOPSIS	Selection of right mask in COVID-19
[53]	Deveci et al.	2021	Type 2 Neutrosophic Sets	MABAC	Site selection of offshore wind farm location
[54]	Nădăban, S. et al.	2016	Neutrosophic Set (NS)	Fuzzy TOPSIS	Supplier selection, sustainable and renewable energy location selection
[55]	Irvanizam et al.	2022	Trapezoidal Fuzzy Neutrosophic Sets	Ordinal Priority Approach (OPA) and MULTIMOORA	Social Aid Distribution Problem
[56]	Bavia et al.	2022	Single Valued Neutrosophic Set (SVNS)	hybrid score accuracy based MCGDM method	Logistics Centre Location Problem
[57]	Abdel-Basset et al.	2022	Single Valued Neutrosophic Set (SVNS)	AD principles based MCDM	Selecting the suitable medical image modality
This paper		2023	Trapezoidal neutrosophic number (TrNN)	NAHP, NTOPSIS & NCOPRAS	Women's university site selection

For further information, you may refer to the recent papers [58–65].

2.4. Site Selection and University Site Selection

Site selection problem is a challenging task as it is based on several multiple conflicting factors (for example [66–70]). For selection of a site, the experts need to consider the long term stability and profitability factor. A comparative study of different types of site selection problem under different methodology is discussed in Table 3.

Table 3. Review of different type of site selection problems with solution strategy.

	Authors	Year	Types of Site	Methodology	Environment
[71]	Wang, C. N. et al.	2018	Renewable energy plants location	FAHP & TOPSIS	Fuzzy
[72]	Wang, C. N. et al.	2018	Solar power plant location	DEA, fuzzy AHP & TOPSIS	Fuzzy
[73]	Maghsoodi, A. I. et al.	2019	Construction project site	BWM, CODAS & T-MADM	Crisp
[74]	Zolfani, S. H. et al.	2022	Different types of site (Review paper)	Different MADM methodology	Fuzzy & crisp
[75]	Kharat, M. G. et al.	2016	Landfill site	Fuzzy AHP & Fuzzy TOPSIS	Fuzzy
[76]	Karaşan, A. et al.	2020	Electric vehicles charging stations	AHP, DEMATEL & TOPSIS	Fuzzy
[77]	Boyacı, A. Ç. et al.	2022	Pandemic hospital location	Fuzzy AHP & TOPSIS	Fuzzy
[78]	Önüt, S. et al.	2010	Shopping center site	AHP & TOPSIS	Fuzzy
[79]	Sennaroglu, B. et al.	2018	Military airport location	AHP, PROMETHEE & VIKOR	Crisp
[80]	Rezaeisabzevar, Y. et al.	2020	Landfill site	AHP, TODIM & ANP	Fuzzy
[81]	Liu, H. C. et al.	2019	Electric vehicle charging stations	DEMATEL & MULTIMOORA	Grey
[82]	Zhou, S. et al.	2012	Biofuel refinery location	Fuzzy TOPSIS	Fuzzy
This paper		2023	Women's university site selection	NAHP, NTOPSIS & NCOPRAS	Neutrosophic

Site selection for educational institutes with some related factors and the procedure of selection have been discussed in [83–85]. In this section, we shall study the present situation of the established universities and institutions in West Bengal, India and try to figure out the problem that forms the basis of the main model in this study.

There are different types of universities and institutes according to their main themes as follows:

1. **Fully research university:** This type of university is primarily focused on research. They have few full-time postgraduate courses but with motivation for research. Also, they have mostly focused Ph.D. programs. The establishment of this type of university needs more funding as the number of students are less.
2. **Fully academic university:** The primary goal of this type of university is academic purposes. They run undergraduate and postgraduate with several specializations. Research might not be the primary focus of this type of university.
3. **Mixed type university (academic and research):** The academic program and research program are both in this type of university. There is a balance between academic and research activity.
4. **Affiliated university:** This type of university is for only controlling the affiliated colleges that are under this university. Different work like inspection of college, control of examination, quality improvement of colleges, etc., plays an important role in this type of university.
5. **University for some special purposes:** University, which is beneficial for the society, country or an organization.

Thus, the above said institutes can be further categorized into:

A **Fully government university**: University which is run by government funding and direction which is fully controlled by the government.

B **Government aided university**: Public and private partnerships with government collaboration. A few portions are helped and directed by the government and the rest is executed by the institute's internal organization.

C **Private university**: Fully funded and directed by the private sector, i.e., non-governmental organization.

Table 4 and Figure 1 represent the university or institution available in the state of West Bengal, in India, in terms of numbers and locations.

Table 4. Different type of university or institution in West Bengal, India [86,87].

Type of University/Institution	Type	Number of University/Institution
Institute of national importance	Fully government	9
Research institutes	Fully government	15
State universities	Government aided	36
Private universities	Private	12
Deemed universities	Government	2
Central universities	Government	1
National law university	Government	1

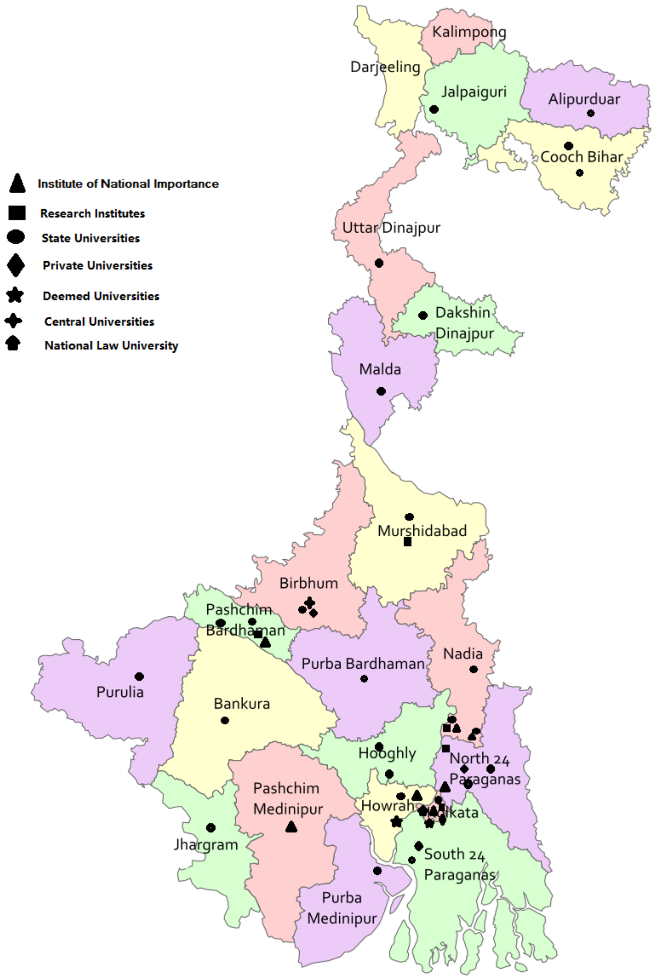

Figure 1. Location wise university & institutions in West Bengal, India [88].

3. Preliminaries

3.1. Neutrosophic Logic

In neutrosophic logic, each proposition is represented by the percentage of occurrence for the truth factor, indeterminacy factor, and falsity factor altogether [35,89].

The range of true membership function/T value, indeterminacy membership function/I value, and falsity membership function/F value are a subset of $]^{-}0, 1^{+}[$ and the supremum of neutrosophic logic and infimum of neutrosophic logic are $n_{sup} = \sup T + \sup I + \sup F \leq 3$ and $n_{inf} = \inf T + \inf I + \inf F \geq 0$ respectively.

Example 1. *Let x(0.7, 0.5, 0.3) be a neutrosophic number with $x \in X$, and the element x is 70% true, 50% indeterminate, and 30% false.*

Example 2. *Consider x((0.7,0.8), (0.5,0.6), (0.25,0.3)) be a neutrosophic number with $x \in X$ and the element x is 70% to 80% true, 50% to 60% indeterminate and 25% to 30% false. Here, we see that the membership value (truth, indeterminacy & falsity) are set with continuous or discrete, closed or open, union or intersection of sets, etc., based on the element/number x.*

3.2. Neutrosophic Sets

Consider three real-valued standard category subsets or non-standard category subsets T, I & I of $]^{-}0, 1^{+}[$ with supremum and infimum defined as true $\sup T = t\_\sup$, $\inf T = t\_\inf$, for indeterminate $\sup I = i\_\sup$, $\inf I = i\_\inf$ and for false $\sup F = f\_\sup$, $\inf F = f\_\inf$. Neutrosophic supremum and neutrosophic infimum are $n_{sup} = \sup T + \sup I + \sup F$ and $n_{inf} = \inf T + \inf I + \inf F$ respectively.

Let us consider the universe of discourse denoted by X with arbitrary subset $A \subset X$. An arbitrary element $y \in X$ belongs to the set A. On the basis of the neutrosophic set, refs. [89–91] denoted as $y(t, i, f)$ and described as follows: y is t% true, i% is indeterminate (undetermined, it may or may not be true) and f% false on the basis of the set A subset of X, where the variables $t \in T, i \in I$ & $f \in F$.

Here, the subsets of $]^{-}0, 1^{+}[$ are T, I & F, are not only fixed set, but are operators/functions calculated on the basis of y and various known/unknown variables. The concept of Neutrosophic sets was given by Florentin Smarandache [35,92].

Definition 1 ([93,94])**.** *Assume X to be a universal set of discourse. Let neutrosophic set N^{+} is presented in the form: $N^{+}(y) = \{< y; T(y), I(y), F(y) : y \in X >\}$, where $T(y)$ is truth, $I(y)$ is indeterminacy and $F(y)$ is falsity functional component of an arbitrary element $y \in X$ with the mapping: $T, I, F : y \rightarrow]0^{-}, 1^{+}[$ and satisfy the following condition: $0 \leq T(y) \leq 1$, $0 \leq I(y) \leq 1$, $0 \leq F(y) \leq 1$ and overall $0 \leq T(y) + I(y) + F(y) \leq 3$.*

3.3. Single Valued Neutrosophic Set (SVNS)

Single Valued Neutrosophic Set (SVNS) [95,96] is a set that consists of element(s) only one element in each membership value. Let $\tilde{\Gamma}(\zeta) = \{\zeta; T_{\Gamma}(\zeta), I_{\Gamma}(\zeta), F_{\Gamma}(\zeta) | \zeta \in \tilde{\Gamma}\}$ be a Neutrosophic Set [97] with each element $\zeta \in \tilde{\Gamma}$ and the true membership functions $T_{\Gamma}(\zeta)$, indeterminacy membership functions $I_{\Gamma}(\zeta)$ and false membership functions $F_{\Gamma}(\zeta)$ are unique value for $\zeta \in \tilde{\Gamma}$ are represented respectively. The range of membership functions lie between $[0, 1]$, i.e., $(0 \leq T_{\Gamma}(\zeta), I_{\Gamma}(\zeta), F_{\Gamma}(\zeta) \leq 1)$.

Example 3. *Let $\tilde{A_1} = \{x; 1, 0.5, 0.6\}$, $\tilde{A_2} = \{x; 0.8, 0.3, 0.5\}$ and $\tilde{A_3} = \{x; 0.7, 0.2, 0\}$ are three SVNSs. Neutrosophic set $\tilde{A_1}$ contains one element x with true membership value 1, indeterminacy membership value 0.5, and false membership value 0.6. Similar rule exists for the set $\tilde{A_2}$ and $\tilde{A_3}$. All these three membership values of the neutrosophic set lie between [0, 1].*

Example 4. *Let $\tilde{B_1} = \{\{y; 0.9, 0.35, 0.42\}, \{z; 0.7, 0.25, 0.15\}\}$ and $\tilde{B_2} = \{x; 1, 0.1, 0.2\}$ are two SVNSs. Here Neutrosophic set $\tilde{B_1}$ consisting of two elements, their three membership values, i.e., true, indeterminacy & falsity membership value of the element $y, z \in \tilde{B_1}$ has exactly one value, i.e.,* *true membership value is 0.9 & indeterminacy and falsity membership values are 0.35 and 0.42 respectively for the element $y \in \tilde{B}_1$ and true membership value is 0.7 & indeterminacy and falsity membership values are 0.25 and 0.15 respectively for the element $z \in \tilde{B}_1$. Similarity true for the neutrosophic set $\tilde{B}_2$, true membership value is 1, and indeterminate and false membership values are 0.1 and 0.2, respectively.*

3.4. Trapezoidal Neutrosophic Number

There are several research papers on neutrosophic numbers and neutrosophic sets. To capture the uncertainties prevailing in real-life situations and to fix the problem, Neutrosophic numbers are more reliable. Different types of developed neutrosophic numbers, such as triangular neutrosophic numbers, trapezoidal neutrosophic numbers [98], pentagonal neutrosophic numbers, and hexagonal neutrosophic numbers, were framed in literature. Considering the problem of this study, TRNNs have been used. The following section includes the definition, arithmetic operations, examples, distance measures, and de-neutrosophication of TRNNs.

Definition 2. *Trapezoidal Neutrosophic Set (TrNS)*

Let $\tilde{\Gamma}(\zeta) = \{(\rho_1, \rho_2, \rho_3, \rho_4; \sigma_1, \sigma_2, \sigma_3, \sigma_4; \phi_1, \phi_2, \phi_3, \phi_4); t_\Gamma(\zeta), i_\Gamma(\zeta), f_\Gamma(\zeta)\}$ be trapezoidal neutrosophic set with ζ be an element on it. Then it's true membership functions $T_\Gamma(\zeta)$, indeterminacy membership functions $I_\Gamma(\zeta)$ and false membership functions $F_\Gamma(\zeta)$ are represented respectively as:

$$T_\Gamma(\zeta) = \begin{cases} 0 & if\ \zeta \leq \rho_1 \\ t_\Gamma \frac{(\zeta-\rho_1)}{\rho_2-\rho_1} & if\ \rho_1 \leq \zeta \leq \rho_2 \\ t_\Gamma & if\ \rho_2 \leq \zeta \leq \rho_3 \\ t_\Gamma \frac{(\rho_4-\zeta)}{\rho_4-\rho_3} & if\ \rho_3 \leq \zeta \leq \rho_4 \\ 0 & if\ \rho_4 \leq \zeta \end{cases} \tag{1}$$

$$I_\Gamma(\zeta) = \begin{cases} 1 & if\ \zeta \leq \sigma_1 \\ \frac{(\sigma_2-\zeta)+i_\Gamma(\zeta-\sigma_1)}{\sigma_2-\sigma_1} & if\ \sigma_1 \leq \zeta \leq \sigma_2 \\ i_\Gamma & if\ \sigma_2 \leq \zeta \leq \sigma_3 \\ \frac{(\zeta-\sigma_3)+i_\Gamma(\sigma_4-\zeta)}{\sigma_4-\sigma_3} & if\ \sigma_3 \leq \zeta \leq \sigma_4 \\ 1 & if\ \sigma_4 \leq \zeta \end{cases} \tag{2}$$

$$F_\Gamma(\zeta) = \begin{cases} 1 & if\ \zeta \leq \phi_1 \\ \frac{(\phi_2-\zeta)+f_\Gamma(\zeta-\phi_1)}{\phi_2-\phi_1} & if\ \phi_1 \leq \zeta \leq \phi_2 \\ f_\Gamma & if\ \phi_2 \leq \zeta \leq \phi_3 \\ \frac{(\zeta-\phi_3)+f_\Gamma(\phi_4-\zeta)}{\phi_4-\phi_3} & if\ \phi_3 \leq \zeta \leq \phi_4 \\ 1 & if\ \phi_4 \leq \zeta \end{cases} \tag{3}$$

here $0 \leq T_\Gamma(\zeta) \leq 1$, $0 \leq I_\Gamma(\zeta) \leq 1$, $0 \leq F_\Gamma(\zeta) \leq 1$ and $0 \leq T_\Gamma(\zeta) + I_\Gamma(\zeta) + F_\Gamma(\zeta) \leq 3$. Then $\tilde{\Gamma}(\zeta) = \{(\rho_1, \rho_2, \rho_3, \rho_4; \sigma_1, \sigma_2, \sigma_3, \sigma_4; \phi_1, \phi_2, \phi_3, \phi_4); t_\Gamma(\zeta), i_\Gamma(\zeta), f_\Gamma(\zeta)\}$ is trapezoidal neutrosophic set (TrNS) when $\rho_1, \rho_2, \rho_3, \rho_4 \in \mathbb{R}$; $\sigma_1, \sigma_2, \sigma_3, \sigma_4 \in \mathbb{R}$; $\phi_1, \phi_2, \phi_3, \phi_4 \in \mathbb{R}$ and $\rho_1 \leq \rho_2 \leq \rho_3 \leq \rho_4$; $\sigma_1 \leq \sigma_2 \leq \sigma_3 \leq \sigma_4$; $\phi_1 \leq \phi_2 \leq \phi_3 \leq \phi_4$.

Definition 3. *Trapezoidal Neutrosophic Set (TrNS)*

Let $\tilde{\Gamma}(\zeta) = \{(\mu_1, \mu_2, \mu_3, \mu_4); t_\Gamma(\zeta), i_\Gamma(\zeta), f_\Gamma(\zeta)\}$ be trapezoidal neutrosophic set with ζ be an element on it. Then it's true membership functions $T_\Gamma(\zeta)$, indeterminacy membership functions $I_\Gamma(\zeta)$ and false membership functions $F_\Gamma(\zeta)$ are represented respectively as:

$$T_\Gamma(\zeta) = \begin{cases} 0 & if\ \zeta \leq \mu_1 \\ t_\Gamma \frac{(\zeta-\mu_1)}{\mu_2-\mu_1} & if\ \mu_1 \leq \zeta \leq \mu_2 \\ t_\Gamma & if\ \mu_2 \leq \zeta \leq \mu_3 \\ t_\Gamma \frac{(\mu_4-\zeta)}{\mu_4-\mu_3} & if\ \mu_3 \leq \zeta \leq \mu_4 \\ 0 & if\ \mu_4 \leq \zeta \end{cases} \tag{4}$$

$$I_\Gamma(\zeta) = \begin{cases} 1 & if\ \zeta \leq \mu_1 \\ \frac{(\mu_2-\zeta)+i_\Gamma(\zeta-\mu_1)}{\mu_2-\mu_1} & if\ \mu_1 \leq \zeta \leq \mu_2 \\ i_\Gamma & if\ \mu_2 \leq \zeta \leq \mu_3 \\ \frac{(\zeta-\mu_3)+i_\Gamma(\mu_4-\zeta)}{\mu_4-\mu_3} & if\ \mu_3 \leq \zeta \leq \mu_4 \\ 1 & if\ \mu_4 \leq \zeta \end{cases} \tag{5}$$

$$F_\Gamma(\zeta) = \begin{cases} 1 & if\ \zeta \leq \mu_1 \\ \frac{(\mu_2-\zeta)+f_\Gamma(\zeta-\mu_1)}{\mu_2-\mu_1} & if\ \mu_1 \leq \zeta \leq \mu_2 \\ f_\Gamma & if\ \mu_2 \leq \zeta \leq \mu_3 \\ \frac{(\zeta-\mu_3)+f_\Gamma(\mu_4-\zeta)}{\mu_4-\mu_3} & if\ \mu_3 \leq \zeta \leq \mu_4 \\ 1 & if\ \mu_4 \leq \zeta \end{cases} \tag{6}$$

here $0 \leq T_\Gamma(\zeta) \leq 1$, $0 \leq I_\Gamma(\zeta) \leq 1$, $0 \leq F_\Gamma(\zeta) \leq 1$ *and* $0 \leq T_\Gamma(\zeta) + I_\Gamma(\zeta) + F_\Gamma(\zeta) \leq 3$. *Then* $\tilde{\Gamma}(\zeta) = \{(\mu_1, \mu_2, \mu_3, \mu_4); t_\Gamma(\zeta), i_\Gamma(\zeta), f_\Gamma(\zeta)\}$ *is trapezoidal neutrosophic set (TrNS) when* $\mu_1, \mu_2, \mu_3, \mu_4 \in \mathbb{R}$ *and* $\mu_1 \leq \mu_2 \leq \mu_3 \leq \mu_4$.

Definition 2 is generalised definition of Trapezoidal Neutrosophic Set (TrNS).

Note: Geometric presentation of trapezoidal neutrosophic number $\tilde{\Gamma}(\zeta) = \{(\beta_1, \beta_2, \beta_3, \beta_4); t_\Gamma(\zeta), i_\Gamma(\zeta), f_\Gamma(\zeta)\}$ are shown in Figure 2 where $\beta_1, \beta_2, \beta_3$ & β_4 are first, second, third and fourth entry of the trapezoidal neutrosophic numbers respectively and $t = t_\Gamma(\zeta)$ for maximum true membership value, $i = i_\Gamma(\zeta)$ for maximum indeterminacy membership value and $f = f_\Gamma(\zeta)$ for maximum false membership value of TrNN $\tilde{\Gamma}(\zeta)$.

Example 5. *Let* $\tilde{\Gamma}(\zeta) = \{(2, 3, 4, 5); 0.7, 0.3, 0.5\}$ *be trapezoidal neutrosophic set with* ζ *be an element on it. Then it's true membership functions* $T_\Gamma(\zeta)$*, indeterminacy membership functions* $I_\Gamma(\zeta)$ *and false membership functions* $F_\Gamma(\zeta)$ *are represented respectively as:*

$$T_\Gamma(\zeta) = \begin{cases} 0 & if\ \zeta \leq 2 \\ 0.7 \times \frac{(\zeta-2)}{3-2} = 0.7 \times (\zeta - 2) & if\ 2 \leq \zeta \leq 3 \\ 0.7 & if\ 3 \leq \zeta \leq 4 \\ 0.7 \times \frac{(5-\zeta)}{5-4} = 0.7 \times (5 - \zeta) & if\ 4 \leq \zeta \leq 5 \\ 0 & if\ 5 \leq \zeta \end{cases} \tag{7}$$

$$I_\Gamma(\zeta) = \begin{cases} 1 & if\ \zeta \leq 2 \\ \frac{(3-\zeta)+0.3\times(\zeta-2)}{3-2} = (3-\zeta) + 0.3 \times (\zeta - 2) & if\ 2 \leq \zeta \leq 3 \\ 0.3 & if\ 3 \leq \zeta \leq 4 \\ \frac{(\zeta-4)+0.3\times(5-\zeta)}{5-4} = (\zeta - 4) + 0.3 \times (5 - \zeta) & if\ 4 \leq \zeta \leq 5 \\ 1 & if\ 5 \leq \zeta \end{cases} \tag{8}$$

$$F_{\Gamma}(\zeta) = \begin{cases} 1 & if\ \zeta \le 2 \\ \frac{(3-\zeta)+0.5\times(\zeta-2)}{3-2} = (3-\zeta)+0.5\times(\zeta-2) & if\ 2 \le \zeta \le 3 \\ 0.5 & if\ 3 \le \zeta \le 4 \\ \frac{(\zeta-4)+0.5\times(5-\zeta)}{5-4} = (\zeta-4)+0.5\times(5-\zeta) & if\ 4 \le \zeta \le 5 \\ 1 & if\ 5 \le \zeta \end{cases} \quad (9)$$

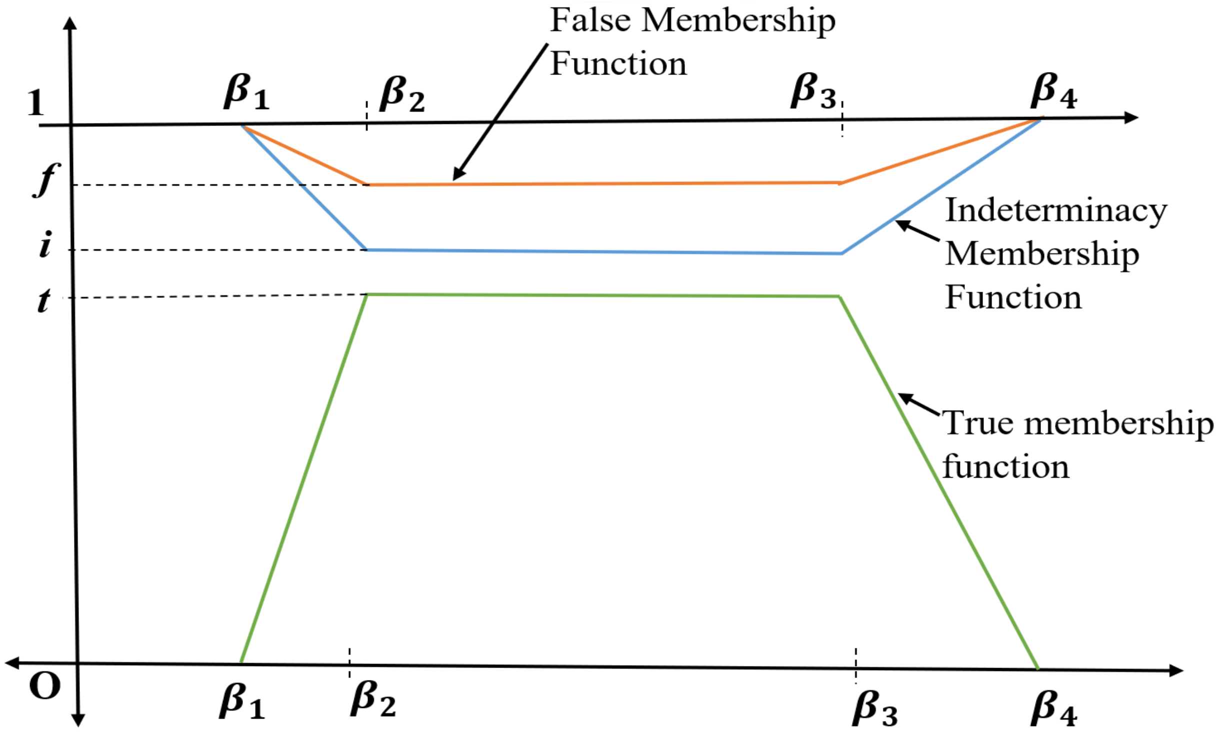

Figure 2. Geometric representation of trapezoidal neutrosophic number (TrNN).

3.5. Arithmetic Operation on TrNN

Let us consider two single-valued trapezoidal neutrosophic numbers (TrNN) $\tilde{\Gamma} = \{(\sigma_1, \sigma_2, \sigma_3, \sigma_4); t_\Gamma, i_\Gamma, f_\Gamma\}$ and $\tilde{\Lambda} = \{(\tau_1, \tau_2, \tau_3, \tau_4); t_\Lambda, i_\Lambda, f_\Lambda\}$ with t_ξ denoted true membership function, i_ξ denoted indeterminacy membership function and f_ξ denoted false membership function where $\xi \in \{\Gamma, \Lambda\}$. Therefore arithmetic operations on TrNN are defined as:

I. **Addition of two TrNNs:**

$$\tilde{\Sigma} = \tilde{\Gamma} \oplus \tilde{\Lambda} = \{(\sigma_1 + \tau_1, \sigma_2 + \tau_2, \sigma_3 + \tau_3, \sigma_4 + \tau_4); t_\Gamma + t_\Lambda - t_\Gamma t_\Lambda,\ i_\Gamma i_\Lambda,\ f_\Gamma f_\Lambda\} \quad (10)$$

II. **Negation of a TrNN:**

$$\tilde{N} = -\tilde{\Gamma} = \{(-\sigma_4, -\sigma_3, -\sigma_2, -\sigma_1); 1 - t_\Gamma, 1 - i_\Gamma, 1 - f_\Gamma\} \quad (11)$$

III. **Subtraction of two TrNNs:**

$$\tilde{\Omega} = \tilde{\Gamma} \ominus \tilde{\Lambda} = \{(\sigma_1 - \tau_1, \sigma_2 - \tau_2, \sigma_3 - \tau_3, \sigma_4 - \tau_4); 1 - t_\Lambda(1 - t_\Gamma), i_\Gamma(1 - i_\Lambda), f_\Gamma(1 - f_\Lambda)\} \quad (12)$$

IV. **Scalar multiplication of a TrNN by *k*:**

$$\tilde{M} = k \times \tilde{\Gamma} = \left\{(k\sigma_1, k\sigma_2, k\sigma_3, k\sigma_4); 1 - (1 - t_\Gamma)^k,\ i_\Gamma^k,\ f_\Gamma^k\right\} \quad (13)$$

V. **Multiplication of two TrNNs:**

$$\tilde{\Pi} = \tilde{\Gamma} \otimes \tilde{\Lambda} = \{(\sigma_1\tau_1, \sigma_2\tau_2, \sigma_3\tau_3, \sigma_4\tau_4); t_\Gamma t_\Lambda,\ i_\Gamma + i_\Lambda - i_\Gamma i_\Lambda,\ f_\Gamma + f_\Lambda - f_\Gamma f_\Lambda\} \quad (14)$$

VI. **Inverse of a TrNN:**

$$\tilde{I} = \tilde{\Gamma}^{-1} = \frac{1}{\{(\sigma_1, \sigma_2, \sigma_3, \sigma_4); t_\Gamma, i_\Gamma, f_\Gamma\}} = \left\{(\frac{1}{\sigma_4}, \frac{1}{\sigma_3}, \frac{1}{\sigma_2}, \frac{1}{\sigma_1}); t_\Gamma, i_\Gamma, f_\Gamma)\right\} \quad (15)$$

3.6. Distance between Two TrNN

Distance measuring between two neutrosophic numbers plays a significant role in MCDM techniques. It gives an idea of ranking of the alternatives. Biswas, P. et al. [99] introduced a distance measure formula as follows:

Definition 4. *(Hamming distance)*
Let $\tilde{\Gamma} = \{(\beta_1, \beta_2, \beta_3, \beta_4); t_\Gamma, i_\Gamma, f_\Gamma\}$ *and* $\tilde{\Lambda} = \{(\tau_1, \tau_2, \tau_3, \tau_4); t_\Lambda, i_\Lambda, f_\Lambda\}$ *are two Trapezoidal Neutrosophic Numbers (TrNN). Then the Hamming distance between* $\tilde{\Gamma}$ *and* $\tilde{\Lambda}$ *is denoted by* $d(\tilde{\Gamma}, \tilde{\Lambda})$ *and defined as:*

$$d(\tilde{\Gamma}, \tilde{\Lambda}) = \left\{ \begin{array}{c} |\beta_1(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_1(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \\ +|\beta_2(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_2(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \\ +|\beta_3(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_3(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \\ +|\beta_4(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_4(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \end{array} \right\} \quad (16)$$

Definition 5. *(Normalized Hamming distance)*
We consider $\tilde{\Gamma} = \{(\beta_1, \beta_2, \beta_3, \beta_4); t_\Gamma, i_\Gamma, f_\Gamma\}$ *and* $\tilde{\Lambda} = \{(\tau_1, \tau_2, \tau_3, \tau_4); t_\Lambda, i_\Lambda, f_\Lambda\}$ *are two trapezoidal neutrosophic numbers (TrNN). Then the normalized Hamming distance between* $\tilde{\Gamma}$ *and* $\tilde{\Lambda}$ *is denoted by* $d_N(\tilde{\Gamma}, \tilde{\Lambda})$ *and defined as:*

$$d_N(\tilde{\Gamma}, \tilde{\Lambda}) = \frac{1}{12} \times \left\{ \begin{array}{c} |\beta_1(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_1(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \\ +|\beta_2(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_2(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \\ +|\beta_3(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_3(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \\ +|\beta_4(2 + t_\Gamma - i_\Gamma - f_\Gamma) - \tau_4(2 + t_\Lambda - i_\Lambda - f_\Lambda)| \end{array} \right\} \quad (17)$$

Remark 1. *In Table 5, we consider two sets of two TrNNs and find the distance between them on the basis of different distance measuring scales. In this study, we consider normalized Hamming distance (given in Equation (17)) as our distance measuring scale.*

Table 5. Distance between two TrNNs by different distance measuring scales.

TrNNs	Euclidean Distance [100–102]	Hausdorff Distance [102]	Chebyshev Distance [100]	Minkowski Distance [100] ($p = 3$)	Normalized Hamming Distance [100–102] & This Paper
$A = \{(2,3,4,5); 0.8, 0.2, 0.3\}$ & $B = \{(4,6,7,9); 0.75, 0.15, 0.2\}$	0.2179449472	0.10	0.01666667	0.0687944616	0.20
$C = \{(0.1, 0.2, 0.3, 0.4); 0.7, 0.25, 0.2\}$ & $D = \{(0.3, 0.6, 0.8, 1); 0.85, 0.15, 0.25\}$	0.0345808227	0.01875	0.003125	0.0329153016	0.0375

3.7. De-Neutrosophication of Neutrosophic Number

Let $\tilde{\Gamma} = \{(\beta_1, \beta_2, \beta_3, \beta_4); t_\Gamma, i_\Gamma, f_\Gamma\}$ be trapezoidal neutrosophic numbers (TrNN) with $0 \leq \beta_1 \leq \beta_2 \leq \beta_3 \leq \beta_4 \leq 1$ and membership functions satisfy $0 \leq t_\Gamma, i_\Gamma, f_\Gamma \leq 1$. Therefore, De-neutrosophication of neutrosophic number $\tilde{\Gamma}$ is symbolized by $\mathscr{N}(\tilde{\Gamma})$ and described as:

$$\mathscr{N}(\tilde{\Gamma}) = \frac{1}{10}(\beta_1 + \beta_2 + \beta_3 + \beta_4) \times (2 + t_\Gamma - i_\Gamma - f_\Gamma) \quad (18)$$

Note: We have constructed the formula in Equation (18) in a new way. Basically, the method stands for the transformation of a trapezoidal neutrosophic number to a crisp number. In a similar way, we may construct de-neutrosophication of other numbers, such as triangular neutrosophic numbers, pentagonal neutrosophic numbers, and hexagonal neutrosophic numbers.

Example 6. *Consider four trapezoidal neutrosophic numbers W, X, Y & Z and they denoted as* $W = \{(3, 4, 6, 7); 0.75, 0.35, 0.3\}$, $X = \{(5, 7, 10, 13); 0.9, 0.4, 0.2\}$, $Y = \{(0.2, 0.4, 0.5, 0.8); 0.8, 0.25, 0.3\}$ & $Z = \{(0.7, 0.8, 0.9, 1); 0.85, 0.1, 0.05\}$. *Then the different de-neutrosophication of TrNNs are shown in Table 6.*

Table 6. De-neutrosophication of trapezoidal neutrosophic number (TrNN).

TrNN	Method 1 [103]	Method 2 [103]	Method 3 [103]	Method 4 [103]	Method 5 [103]	Method 6 [103]	Method 7 [103]	Above Method (18)
W	3.6875	7	3.5	3.5	3.5	0.8333	7.5	4.2
X	7.8433	13	6.7083	6.6444	6.7374	0.2778	3.9	8.05
Y	0.3949	0.63	0.3563	0.35	0.3607	0.6403	0.175	0.4275
Z	0.6524	0.85	0.765	0.765	0.765	0.5344	0.8925	0.918

Remark 2. *De-neutrosophication of TrNNs is shown in Table 6, and methodology is described in Section 3.7 (Equation (18)). Consider four TrNN are W, X, Y & Z and find the corresponding de-neutrosophic number of them on the basis of different methods described in [103,104] and in this paper.*

4. Used Multi-Criteria Decision Making Methods

The MCDM methods AHP in the neutrosophic environment, TOPSIS in the neutrosophic environment, and COPRAS in the neutrosophic environment are introduced in this section. The NAHP is used to check the consistency of the decision matrix. The trapezoidal neutrosophic number for criteria weight is computed by the proposed model in the Section 4.2. The ranking of different alternatives was done by NTOPSIS and NCOPRAS method. Graphical representation of complete ranking flowchart figure shown in Figure 3.

4.1. Analytic Hierarchy Process (AHP)

Thomas L. Saaty [105] introduce AHP in the year 1980, which is one of the most familiar MCDM techniques. This method considers the mutual association between the criteria. In real-life situations, the criterion's are dependent, and thus while decision-making, the mutual relationship existence of criterion's should be taken in account. AHP is helpful for handling various qualitative and quantitative multi-criteria factors involving complicated decision-making problems. AHP algorithm works on the principle of construction of pairwise comparison matrix, based on their relative importance. This technique makes decision and calculation simple because of the delight analogy.

Considering ten criterion's and six alternatives to select the best site for women university. The steps of NAHP methods [106–108] are as follows:

I. Recognition and study of the criterion's and their respective sub-criterion's.

II. On the basis of opinions of DMs, construction of a pairwise comparison matrix with the trapezoidal neutrosophic numbers (TrNNs). Let N number of DMs give their decision. Individual DM reveal their own view in terms of the pairwise comparison matrix of criterion's. Let us assume t number of criterion's, then, the comparison matrix has ordered $t \times t$ square matrix. Now N set of matrices are obtained $D_c = \{d_{ijc}\}$ where $c = 1, 2, \ldots, N$ & $i, j = 1, 2, \ldots, t$.

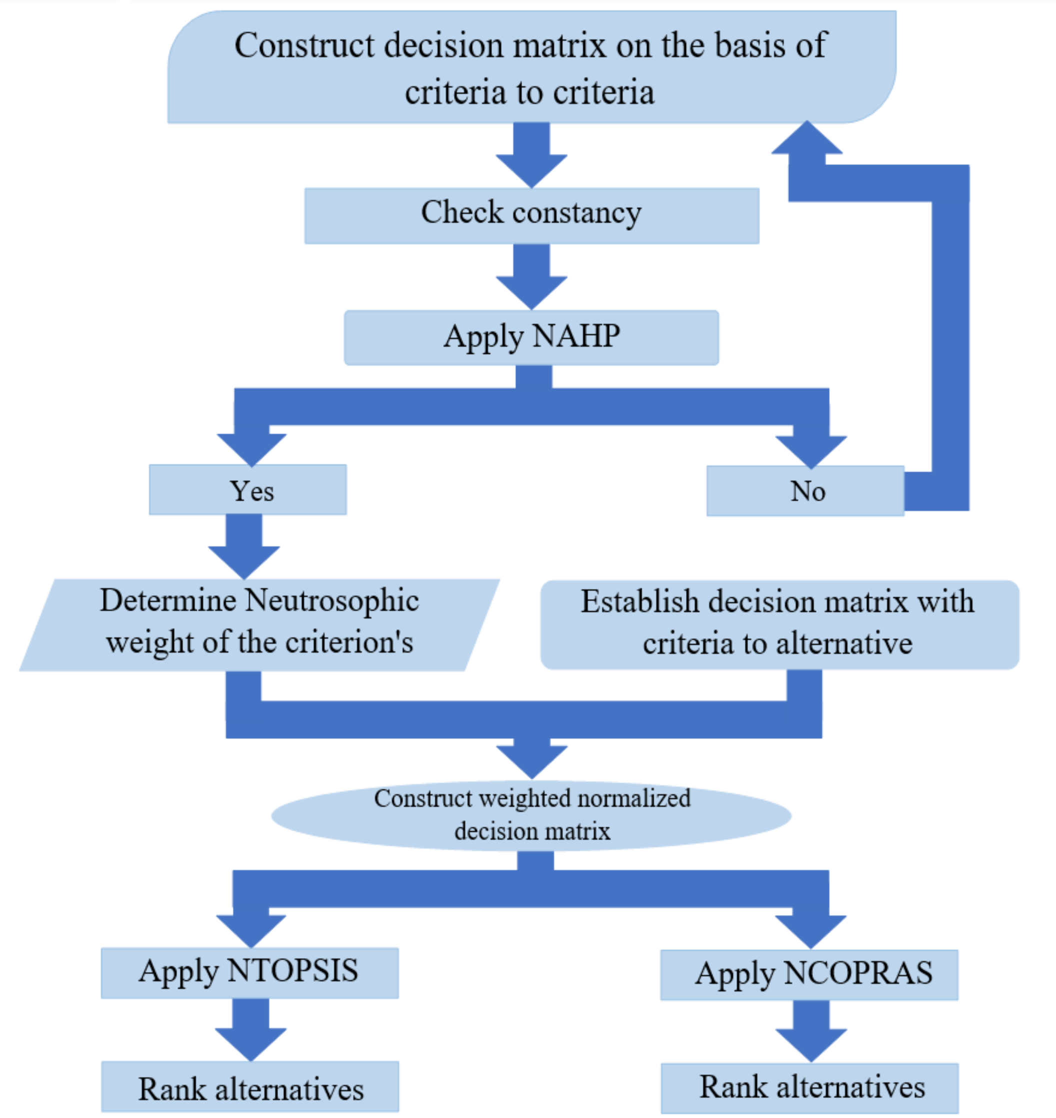

Figure 3. Diagrammatic structure of finding the best site of the women's university.

Now $d_{ijc} = \{(\alpha_{ijc}, \beta_{ijc}, \gamma_{ijc}, \delta_{ijc}); t_{ijc}, i_{ijc}, f_{ijc}\}$ indicate TrNN of i criteria to j criteria as communicate by the DM 'c'.

$$\begin{cases} \alpha_{ij} = \min_{c=1,2,\ldots,N} \alpha_{ijc} \\ \beta_{ij} = \sqrt[N]{\prod_{c=1}^{N} \beta_{ijc}} \\ \gamma_{ij} = \sqrt[N]{\prod_{c=1}^{N} \gamma_{ijc}} \\ \delta_{ij} = \max_{c=1,2,\ldots,N} \delta_{ijc} \\ t_{ij} = \min_{c=1,2,\ldots,N} t_{ijc} \\ i_{ij} = \max_{c=1,2,\ldots,N} i_{ijc} \\ f_{ij} = \max_{c=1,2,\ldots,N} f_{ijc} \end{cases} \tag{19}$$

III. De-neutrosophication of TrNN:
De-neutrosophication of the TrNN is done by using Equation (18) of the matrix

$$A_{t\times t} = [a_{ij}]_{t\times t} \tag{20}$$

where $i, j = 1, 2, \ldots, t$.

IV. Normalization of De-neutrosophication matrix:
Normalized the comparison matrix A get from Equation (20) and converted into matrix B. Each entry of B is evaluated as

$$b_{ij} = \frac{a_{ij}}{\sum_{k=1}^{t} a_{kj}} \tag{21}$$

Note: The weighted sum of each column on the normalization matrix is equal to one.

V. Evaluation of criteria weights:
Criteria weight w_j of each criteria j is determined using

$$w_j = \frac{\sum_{k=1}^{t} b_{kj}}{t} \tag{22}$$

VI. Determination of weight sum value and λ_{max}:
The weight sum value of each criterion j is

$$w'_j = \sum_{k=1}^{t} (a_{ij} \times w_j) \tag{23}$$

then λ_{max} is

$$\lambda_{max} = \frac{1}{t} \times \frac{w'_j}{w_j} \tag{24}$$

VII. Calculation of Consistency Index (CI):
The consistency index (CI) of the matrix is estimated. CI of the matrix is computed using the following:

$$CI = \frac{\lambda_{max} - t}{t - 1} \tag{25}$$

where t denotes the number of criteria, which ultimately represents the size of the matrix.

VIII. Finally, the consistency ratio (CR) is calculated.

$$CR = \frac{CI}{RI} \tag{26}$$

where random index (RI) is the standardised. The values of RI varies with respect to the order of the matrix (i.e., the number of criterion). The size of the matrix and corresponding values of RI are shown in Table 7 (values of t vary 1 to 10).

Table 7. Random Index (RI) value for different size of comparison matrix (t) by Saaty [105].

Matrix Size (t)	1	2	3	4	5	6	7	8	9	10
Random Index (RI)	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

The computation of $CR \leq 0.1$ is acceptable and indicates that the weights obtained are consistent.

4.2. Determination of Trapezoidal Neutrosophic Numbers (TrNNs) Weights of Criterion's

In this study, we find an innovative way of finding trapezoidal neutrosophic numbers (TrNNs) weights of criterion. This neutrosophic weight is used in Section 4.3 for NTOPSIS method and Section 4.4 for NCOPRAS method. The computational procedures are as follows:

(a) Recognition and study of the criterion for MCDM. Construction of pairwise comparison matrix in terms of TrNNs given by the DMs.
(b) Aggregation of the opinions of 'N' DMs using the operator followed by the Equation (19).
(c) The geometric mean is calculated of the TrNN in the comparison matrix by using

$$\{(\alpha_j, \beta_j, \gamma_j, \delta_j), t_j, i_j, f_j\} = \left\{\left((\prod_{i=1}^{t}\alpha_{ij})^{\frac{1}{t}}, (\prod_{i=1}^{t}\beta_{ij})^{\frac{1}{t}}, (\prod_{i=1}^{t}\gamma_{ij})^{\frac{1}{t}}, (\prod_{i=1}^{t}\delta_{ij})^{\frac{1}{t}}\right), \min_{i=1}^{t} t_{ij}, \max_{i=1}^{t} i_{ij}, \max_{i=1}^{t} f_{ij}\right\} \quad (27)$$

(d) Addition of trapezoidal numbers and optimization of the membership numbers using the equation

$$\{(\alpha^+, \beta^+, \gamma^+, \delta^+), t^+, i^+, f^+\} = \left\{\left(\sum_{j=1}^{t}\alpha_j, \sum_{j=1}^{t}\beta_j, \sum_{j=1}^{t}\gamma_j, \sum_{j=1}^{t}\delta_j\right), \min_{j=1}^{t} t_j, \max_{j=1}^{t} i_j, \max_{j=1}^{t} f_j\right\} \quad (28)$$

(e) Calculation of the inverse of TrNN getting from the Equation (15) as follows

$$\{(\alpha^-, \beta^-, \gamma^-, \delta^-), t^-, i^-, f^-\} = \left\{\left(\frac{1}{\delta_j}, \frac{1}{\gamma_j}, \frac{1}{\beta_j}, \frac{1}{\alpha_j}\right), t_j, i_j, f_j\right\} \quad (29)$$

(f) The TrNN weights of the criteria is calculated by the equation

$$\{(\alpha_j^w, \beta_j^w, \gamma_j^w, \delta_j^w), t_j^w, i_j^w, f_j^w\} = \left\{(\alpha_j \times \alpha^-, \beta_j \times \beta^-, \gamma_j \times \gamma^-, \delta_j \times \delta^-), min\{t_j, t^-\}, max\{i_j, i^-\}, max\{f_j, f^-\}\right\} \quad (30)$$

Finally, the TrNN weight of the criterion's is obtained from the Equation (30).

4.3. The Neutrosophic Technique for Order of Preference by Similarity to Ideal Solution (NTOPSIS)

Hwang and Yoon [109] proposed MCDM method in the year 1981, which is well known as TOPSIS method. This methodology is imposed on different fields nowadays, such as engineering and manufacturing sector [110], chemical engineering [111], medicine [112], energy [113], water resources studies [114], site selection [115,116], safety and environmental field [117]. In this technique, the decision matrix is created in linguistic terms which are assigned by decision experts.

These linguistic ratings are then transformed to TrNNs [99,118]. The TrNNs are standardized, and then the PIS and NIS are computed for each alternative separately. The concept of this MCDM tool is based on measuring distance in which the optimal alternative is calculated by the nearest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). Finally, for each alternative relative closeness (RC) is measured. The higher value of RC depicts the most optimal alternative. The procedure of NTOPSIS method is as follows:

I. Decision matrices are constructed on the basis of DMs in linguistic ratings. The linguistic assignments are then transformed into TrNNs.
II. Aggregation of the opinions of '*N*' DMs using the operator follow by the Equation (19).
III. Standardization of the TrNNs, using the formula:

$$\tilde{S} = [S_{ij}]_{k\times l} \quad (31)$$

where $i = 1, 2, 3, \ldots, k$; $j = 1, 2, 3, \ldots, l$;
$S_{ij}^{B} = \left\langle\left(\frac{\alpha_{ij}}{\delta_j^+}, \frac{\beta_{ij}}{\delta_j^+}, \frac{\gamma_{ij}}{\delta_j^+}, \frac{\delta_{ij}}{\delta_j^+}\right), t_{ij}, i_{ij}, f_{ij}\right\rangle$; $\delta_j^+ = \max \delta_{ij}$ and $i \in$ Beneficent Criteria (B.C)
and $S_{ij}^{N.B} = \left\langle\left(\frac{\alpha_j^-}{\delta_{ij}}, \frac{\alpha_j^-}{\gamma_{ij}}, \frac{\alpha_j^-}{\beta_{ij}}, \frac{\alpha_j^-}{\alpha_{ij}}\right), t_{ij}, i_{ij}, f_{ij}\right\rangle$; $\alpha_j^- = \min \alpha_{ij}$ and $i \in$ Non-beneficent Criteria (N.B.C).

IV. Weighted standardized matrix is determined by the product of criteria's TrNNs weight (wc) and standardized TrNNs value (S_{ij}).

$$\tilde{WS} = [wcS_{ij}]_{k \times l};\ i = 1, 2, 3, \ldots, k \text{ and } j = 1, 2, 3, \ldots, l \tag{32}$$

where $TrNNW_{ij} = S_{ij} \times wc_j;\ i = 1, 2, 3, \ldots, k$ and $j = 1, 2, 3, \ldots, l$.
For the product of two TrNNs, please follow Equation (14). Trapezoidal neutrosophic numbers (TrNNs) weights of criteria's calculated by the Equation (30).

V. Determination of TrNNs positive ideal solution (TP^+) and TrNNs negative ideal solution TN^-. Here p_i^+ signifies the maximum value of p_{ij} and p_i^- denotes the minimum value of p_{ij}.
$(TP^+) =$
$\langle (r_1^+, \max_i t_{ij}, \min_i i_{ji}, \min_i f_{ij}), (r_2^+, \max_i t_{ij}, \min_i i_{ji}, \min_i f_{ij}), \ldots,$
$(r_l^+, \max_i t_{ij}, \min_i i_{ji}, \min_i f_{ij}) \rangle$

$$= \{(\max p_{ij} | j \in B.C), (\min p_{ij} | j \in N.B.C)\} \tag{33}$$

$(TN^-) =$
$\langle (r_1^-, \min_i t_{ij}, \max_i i_{ji}, \max_i f_{ij}), (r_2^-, \min_i t_{ij}, \max_i i_{ji}, \max_i f_{ij}), \ldots,$
$(r_l^-, \min_i t_{ij}, \max_i i_{ji}, \max_i f_{ij}) \rangle$

$$= \{(\min p_{ij} | j \in B.C), (\max p_{ij} | j \in N.B.C)\} \tag{34}$$

where

$$\begin{cases} r_j^+ = \{(r_j^{+1}, r_j^{+2}, r_j^{+3}, r_j^{+4})\} = \{\max_i(r_{ij}^1), \max_i(r_{ij}^2), \max_i(r_{ij}^3), \max_i(r_{ij}^4)\} \\ r_j^- = \{(r_j^{-1}, r_j^{-2}, r_j^{-3}, r_j^{-4})\} = \{\min_i(r_{ij}^1), \min_i(r_{ij}^2), \min_i(r_{ij}^3), \min_i(r_{ij}^4)\} \end{cases} \tag{35}$$

VI. Relative distance is calculated for each alternative in term of TrNNs (i.e., from positive ideal solution (TP^+) and negative ideal solution (TN^-) respectively).

$$\begin{cases} DP_j^+ = \sum_{j=1}^{l} d(p_{ij}, p_i^+), i = 1, 2, \ldots, k \\ DP_j^- = \sum_{j=1}^{l} d(p_{ij}, p_i^-), i = 1, 2, \ldots, k \end{cases} \tag{36}$$

where DP_j^+ & DP_j^- denotes the Hamming distance. The distance measure used here is given in Equation (17).

VII. Finally, calculation of relative closeness of the alternatives

$$R_j = \frac{DP_j^-}{DP_j^+ + DP_j^-} \tag{37}$$

Ultimately, the ranking of the alternatives by the obtained value of R_j. The higher value of R_j denotes the optimal alternative.

Remark 3. *In this paper, the criteria investment costs* $(\tilde{m}_2)$ *is only non-beneficiary criteria (NBC), and all other criteria are beneficiary criteria (BC). Beneficiary criteria (BC) are those criteria that are beneficial for selectors, and non beneficiary criteria (NBC) are the ones whose declination is beneficial for selectors.*

4.4. Neutrosophic Complex Proportional Assessment (NCOPRAS) Approach

Zavadskas, Kalklauskas, and Sarka [119] first introduced COPRAS in the year 1994. An extended representation of COPRAS is Fuzzy COPRAS which is used for the ordering of the alternatives in various decision-making problems [120]. This method is based on stepwise ranking and evaluation of the alternative in reference to utility degree and significance. Earlier, COPRAS method was applied by Ghosh, A. et al. in electric vehicle charging station site selection [116], Seker, S. in solar power plants site selection [121], Fouladgar, M.M. et al. in property management [122], economy by Narayanamoothy, S. et al. [123], Evaluating the potential capability of air cargo sector Tolga, A.C. and Durak, G. [124], selection of optimal material for the solar car by Ghose, D. et al. [125]. An application of location selection by Bausys, R. et al. [126] using Neutrosophic COPRAS method. The approach of Neutrosophic COPRAS (NCOPRAS) method is as following steps:

i. Establish of TrNN comparison matrix by the expert of decision makers. The DMs allocate in linguistic terms depending on criterion's.
ii. integration of the decision matrix of the viewpoint of 'N' DMs using the operator followed by the Equation (19).
iii. Standardization of decision matrix is computed in the same way as computed in the TOPSIS method using Equation (31).
iv. Construction of weighted standardization decision matrix which is done by product of the TrNN criteria weight and standardization decision matrix.
v. Calculation of beneficiary criteria (BC) and non-beneficiary criteria (NBC) denoted as BC^+ and NBC^- respectively as follows:

$$BC^+ = \left\{ \sum_{j\in B.C} \alpha_{ij}, \sum_{j\in B.C} \beta_{ij}, \sum_{j\in B.C} \gamma_{ij}, \sum_{j\in B.C} \delta_{ij}, \min_{j\in B.C} t_{ij}, \max_{j\in B.C} i_{ij}, \max_{j\in B.C} f_{ij} \right\}$$
(38)
$$NBC^- = \left\{ \sum_{j\in N.B.C} \alpha_{ij}, \sum_{j\in N.B.C} \beta_{ij}, \sum_{j\in N.B.C} \gamma_{ij}, \sum_{j\in N.B.C} \delta_{ij}, \max_{j\in N.B.C} t_{ij}, \min_{j\in N.B.C} i_{ij}, \min_{j\in N.B.C} f_{ij} \right\}$$

where $\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \delta_{ij}, t_{ij}, i_{ij}$ & f_{ij} $(i = 1,2,3,\ldots,k$ and $j = 1,2,3,\ldots,l)$ come from the weighted standardization matrix in Equation (32).

vi. Now, de-neutrosophication of the TrNN using the Equation (18). De-deutrosophication of the Beneficiary Criteria (BC) is denoted by S_i^+, and de-neutrosophication of the non-beneficiary criteria (NBC) is denoted by S_i^-.
vii. Calculate

$$Q_i = S_i^+ + \frac{S_{min}^- \times \sum_{i=1}^{k} S_i^-}{S_i^- \times \sum_{i=1}^{k} \left(\frac{S_{min}^-}{S_i^-} \right)} \tag{39}$$

where $S_{min}^- = min\{S_i^- : i = 1,2,3,\ldots,k\}$ and i indicate as alternatives.

viii. Calculation of the value of R_i

$$R_i = \frac{Q_i}{Q_{max}} \times 100\% \tag{40}$$

where $Q_{max} = \{Q_i : i = 1,2,3,\ldots,k\}$.
Finally, ranking of the alternative in ascending order on the R_i score.

4.5. Pseudo Code Depicting the Empirical Study Application

This study model was constructed with k number of criteria and l number of alternatives with N decision makers (DM). The input variable is given by DMs in the form of linguistic terms. The linguistic terms transform to TrNN to get the output, i.e., ranking alternatives on the process of the neutrosophic MCDM method. The comparison matrix is $k \times k$ matrix form, and the decision matrix is $l \times k$ matrix form.

INPUT: Comparison matrix & Decision matrix
OUTPUT: Ranking the alternatives
COMPUTE: Consistency ratio, weight of the criteria in TrNN
INITIALIZE: TrNN
OPERATION: NAHP, weight in TrNN, NTOPSIS & NCOPRAS

1. **FOR NAHP**
2. **MERGE** merge the N number of DMs inputs of comparison matrix
3. **IF** comparison matrix is inconsistent ($CR \geq 0.1$)
4. **THEN** reconstruct the comparison matrix
5. **ELSE** comparison matrix is consistent ($CR < 0.1$)
6. **END FOR**
7. **COMPUTE** TrNN weight of the criteria
8. **CONSTRUCT** comparison matrix
9. **THEN** compute the weighted normalized comparison matrix
10. **FIND** determine the weight of the criteria in TrNN
11. **THEN** consider the N decision matrix given by DMs
12. **MERGE** merge the N number of DMs inputs of decision matrix
13. **COMPUTE** determination of the weighted normalised decision matrix
14. **BEGIN NTOPSIS**
15. **COMPUTE** calculation of the ranking of the alternatives using weighted normalized decision matrix
16. **END NTOPSIS**
17. **BEGIN NCOPRAS**
18. **COMPUTE** calculation of the ranking of the alternatives using weighted normalized decision matrix
19. **END NCOPRAS**

5. Model for Setting Up a Government/Government Aided University for Women (Selection of the Alternatives)

In backward /under privileged area people's perception towards the co-educational universities and women universities are different. Poor families with their conservative mentality possess inhibitions while sending their girl child to co-educational universities as they have concerns about safety, security and family honour. Hence many attributes which are not so important for co-educational universities are more pertinent for women university. Considering the real-life problem of selecting a university site in the state of eastern India, namely West Bengal, which has been chosen for the current study. From [86,87], the references indicates that there is been only two women's universities currently in this state. So, this study focuses on this fact and thus, site selection for upcoming women's Universities has been chosen for this research. To achieve this goal, identification and evaluation of criteria's are conducted by the expert opinion. Further, sites (headquarter of some district) which are satisfying the needs of the criteria are selected. Furthermore, two criteria's are considered and well thought keeping in mind about women university which are safety and sex ratio. Criteria weight's are computed using the FAHP approach and then ranking of the sites are executed using MCDM tool TOPSIS and COPRAS.

Six locations in different districts of West Bengal are selected for the setup of a government/government-aided women's university. Presently in West Bengal, India, there is two women's university. The first one is "Diamond Harbour Women's University" in Diamond Harbour, South 24 Parganas district, and another is "Kanyashree University" in Krishnanagar, Nadia district. According to the 2011 census, [127] population of West Bengal is 91,276,115, where the female population is 44,467,088, and the male population is 46,809,027. The literacy rate for females is 70.54% and for males 81.69% and overall 76.36%. Female literacy is 10.93% more than the 2001 census. Six locations are considered as alternatives for choosing the best site for women university; Table 8 give their details and Figure 4 shows the position in West Bengal, India's map.

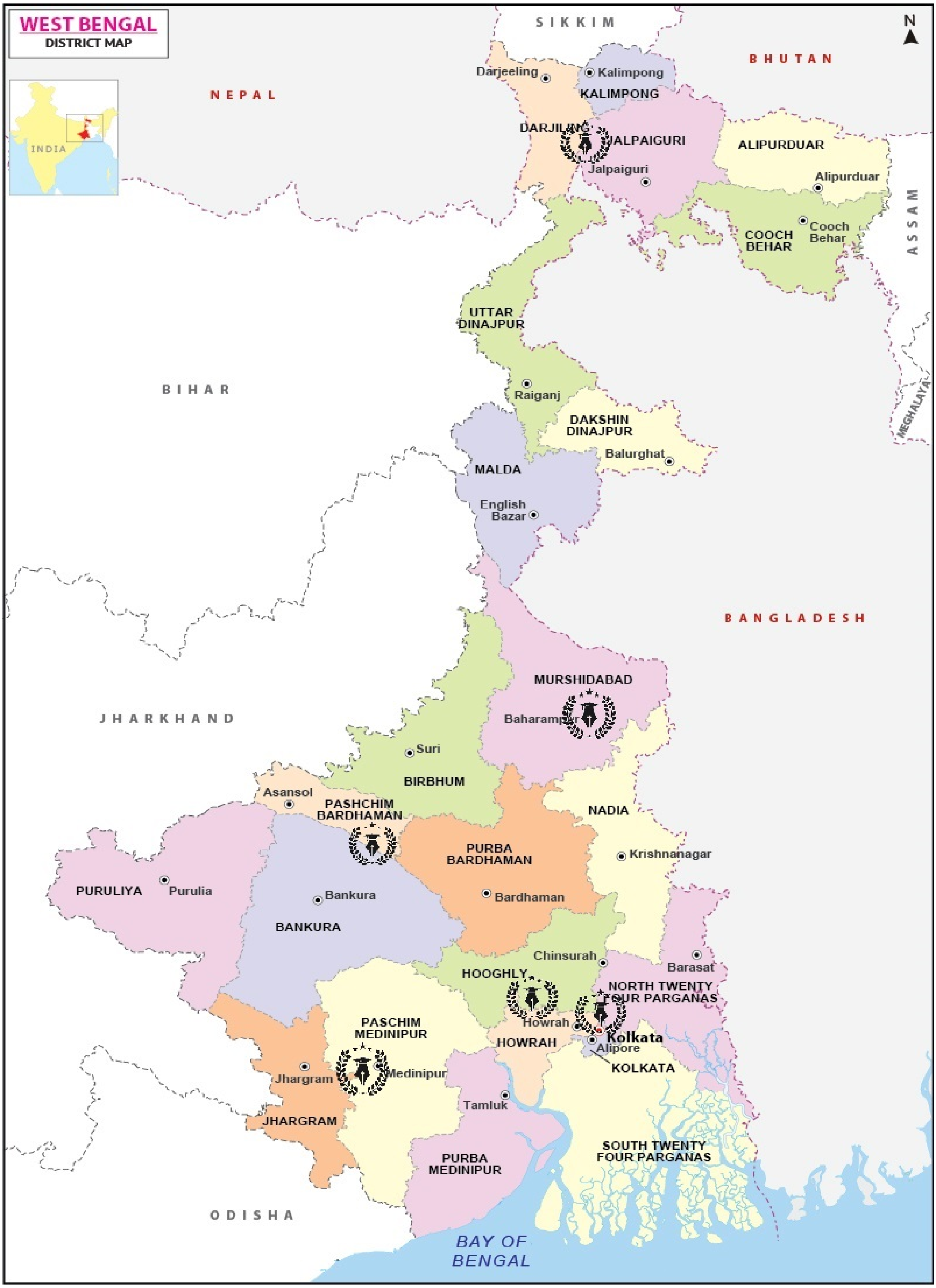

Figure 4. Used locations for Women's University in West Bengal, India [128].

Table 8. Used location (alternative) details.

Location	District	Latitude & Longitude	Location Details
Kolkata (L_{A1})	kolkata	22.5726 °N, 88.3639 °E	It is the capital city of West Bengal.
Howrah (L_{A2})	Howrah	22.5958 °N, 88.2636 °E	It is the capital city of Howrah district.
Berhampore (L_{A3})	Murshidabad	24.0983 °N, 88.2684 °E	It is the capital city of Murshidabad district.
Siliguri (L_{A4})	Darjeeling	26.7271 °N, 88.3953 °E	This city is in Darjeeling district base of the Himalaya mountains and the side of Mahananda river. It is second largest city of West Bengal.
Midnapore (L_{A5})	Paschim Medinipur	22.4257 °N, 87.3199 °E	This city beside the Kangsabati river and capital city of Midnapore district.
Durgapur (L_{A6})	Paschim Bardhaman	23.5204 °N, 87.3119 °E	It is the capital city of Burdwan district.

6. Criteria of Women's University Site Selection

In this section, we are focusing on the different criteria for women's university site selection. After consulting a few experts, we have finalized the following ten criterion. The hierarchical data structure of the Women University site selection is shown in Figure 5.

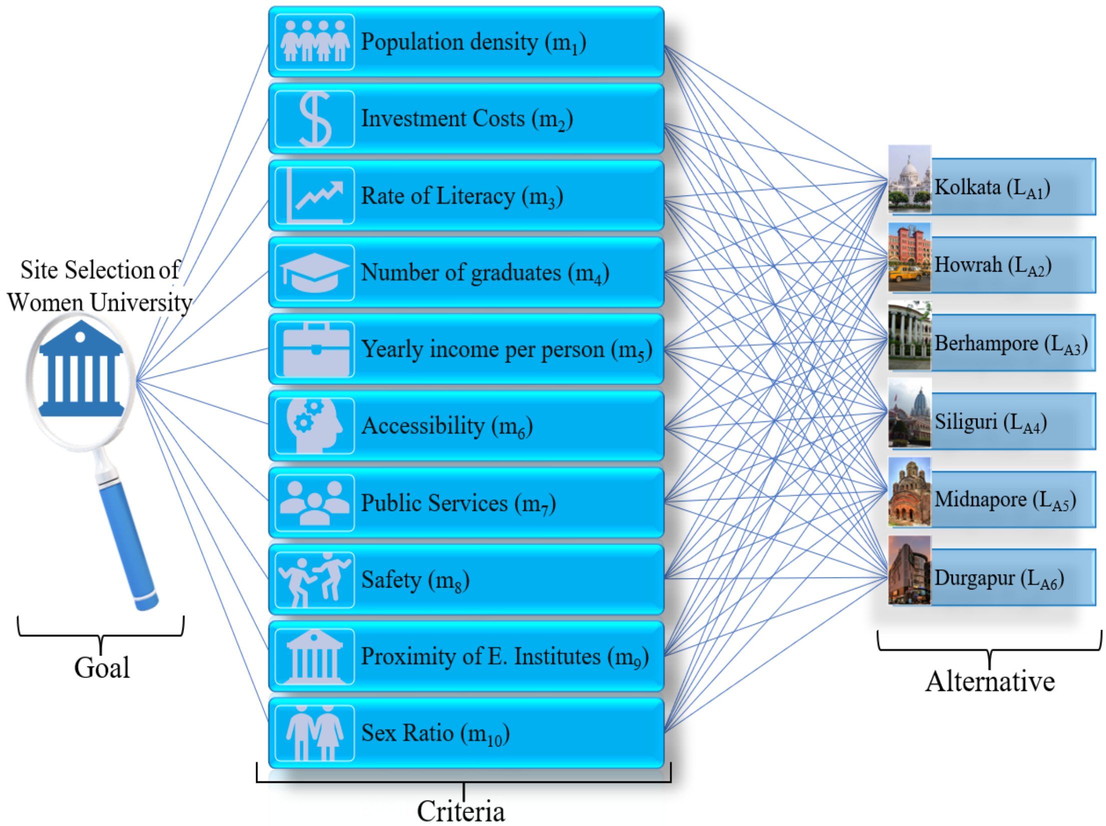

Figure 5. The Hierarchical structure of Women's University site selection problem.

6.1. Population Density ($\tilde{m}_1$) (Fixed Data)

Population density means the number of individuals per unit geographic area. It is commonly expressed in per square km. The large population density of a specific zone implies a greater probability of students enrolling for higher studies. The probability becomes obvious, if the literacy rate of the related area is higher.

6.2. Investment Costs ($\tilde{m}_2$)

Investment cost implies the aggregate cost connected with the project of selecting a site for University construction. This cost includes the price of the land, construction costs, operating costs, and management costs.

6.3. Rate of Literacy ($\tilde{m}_3$) (Fixed Data)

Literacy means the ability of a person in regard to knowledge, optimal decision, behavior, confidence, intellect, rational thinking, communication, etc. Literacy builds a person more willing to acquire or aspire to higher goals. Thus the rate of Literacy is an important attribute when one thinks about the construction of higher educational institutions.

UNESCO, 2010 answers, "Why Is Literacy Important?" in an efficient way, i.e., "Literacy is a human right, a tool of personal empowerment and a means for social and human development. Educational opportunities depend on Literacy. Literacy is at the heart of basic education for all and essential for eradicating poverty, reducing child mortality, curbing population growth, achieving gender equality, and ensuring sustainable development, peace, and democracy".

6.4. Number of Graduates ($\tilde{m}_4$)

This criterion implies the number of the person who has completed their graduation degree and is interested in higher studies. If the number of graduates in a particular location is more, then the possibility of University construction in that specific site increases. The potentiality of a person to study at university is directly proportional to his/her graduate degree.

6.5. Yearly Income per Person ($\tilde{m}_5$)

Income and education have a strong relationship. Higher education leads to multiple professions with good income and vice versa. The person with higher education has low average unemployment than those with less or no education. Thus, data on yearly income per person in the area strongly signifies a greater number of students who can pursue higher studies.

6.6. Accessibility ($\tilde{m}_6$)

Accessible facilities include road transport, i.e., accessibility to vehicles such as trains, buses, and premises accommodation/ hostels for professors, students, and non-teaching staff are quintessential for University site selection.

6.7. Public Services ($\tilde{m}_7$)

Fire safety, such as fire alarms and fire devices, is important to avoid fire dangers in educational institutions. Fire disasters can be prevented if the institutes religiously follow righteous fire safety measures.

6.8. Safety ($\tilde{m}_8$)

Safety for the students and Professors is equally important. Places where social hazards in the neighborhood, such as high incidence of crime and drug or alcohol abuse, are not to be considered safe for University construction. For Women University Site selection, the literacy rate in that area, male/female, is considered significant.

6.9. Proximity of Educational Institutes ($\tilde{m}_9$)

Sites that are selected close to libraries, educational institutions/research institutes are preferred more than those sites which don't have these facilities nearby.

6.10. Sex Ratio ($\tilde{m}_{10}$) (Fixed Data)

The sex ratio or gender ratio is the ratio of females to males in a population. Since this research is about women's University site selection, this factor is quite significant for finding out the optimal site.

7. Data Collection

Data collection for the various sites with respect to different criteria was obtained from the government portal. However, data for every year was not available for all criteria. Thus, the past records were analyzed, and a scale was made corresponding to which the linguistic terms were considered. In criteria-to-criteria comparison, linguistic rating to TrNN transformation by DMs are shown in Table 9. We also give Table 10 for related criterion's with their units and data collection procedure. Researchers may consider another TrNN too. The same concept is applicable in Table 11 where the linguistic rating for criteria to alternative with TrNN is given. The fixed data scaling is also linguistic conversion is performed in our own way (see Tables 12 and 13). Without the fixed data cases, all the linguistic ratings for criteria to criteria and criteria to an alternative are shown in Tables 14 and 15, respectively.

Table 9. Linguistic terms and their corresponding TrNN.

Linguistic Terms	Trapezoidal Neutrosophic Numbers (TrNN)
Equally Important (EI)	$\{(0.4, 0.5, 0.6, 0.7); 0.85, 0.20, 0.15\}$
Moderately Important (MI)	$\{(0.5, 0.6, 0.7, 0.8); 0.85, 0.15, 0.10\}$
Strongly Important (SI)	$\{(0.6, 0.7, 0.8, 0.9); 0.90, 0.15, 0.10\}$
Very Strongly Important (VSI)	$\{(0.7, 0.8, 0.9, 0.95); 0.90, 0.10, 0.05\}$
Absolutely Important (AI)	$\{(0.8, 0.9, 0.95, 1.0); 0.95, 0.10, 0.00\}$
Moderately Not Important (MNI)	$\{(0.3, 0.4, 0.5, 0.6); 0.80, 0.20, 0.15\}$
Strongly Not Important (SNI)	$\{(0.2, 0.3, 0.4, 0.5); 0.80, 0.25, 0.15\}$
Very Strongly Not Important (VSNI)	$\{(0.1, 0.2, 0.3, 0.4); 0.80, 0.25, 0.20\}$
Absolutely Not Important (ANI)	$\{(0.0, 0.1, 0.2, 0.3); 0.75, 0.30, 0.20\}$

Table 10. Related criterion's with their units and data sources.

Serial No	Criteria	Scales (Units)	Source of Data
1	Population density ($\tilde{m}_1$)	This is calculated by average number of population per square kilometer.	This is fixed data collected from Census2011 [129].
2	Investment Costs ($\tilde{m}_2$)	In Rupees.	After considering the investment costs different location.
3	Rate of Literacy ($\tilde{m}_3$)	Divide the number of literates of a given age range by the corresponding age group population and then multiply the result by 100.	This is fixed data collected from Census2011 [129].
4	Number of graduates ($\tilde{m}_4$)	Number of people.	Based on literacy rate.
5	Yearly income per person ($\tilde{m}_5$)	In Rupees.	After considering the average income of the district where the sites are located.
6	Accessibility ($\tilde{m}_6$)	Linguistics term (good, bad etc.).	After seeing the transport and related system of the sites.
7	Public Services ($\tilde{m}_7$)	Linguistics term.	After seeing whether there is public service active or not nearby the sites.
8	Safety ($\tilde{m}_8$)	Linguistics term	After analysing the crime agents women and crime rate nearby the sites.
9	Proximity of E. Institutes ($\tilde{m}_9$)	Linguistics term	After seeing nearby educational institutes exist or not.
10	Sex Ratio ($\tilde{m}_{10}$)	Number of women per 1000 men.	This is fixed data collected from Census2011 [129].

Remark 4. *The linguistic rating by TrNN is given in the above Table 9. This numerical rating is done by us in a scientific manner. Maybe anyone can modify it in their own way but need to follow the ideology of linguistic rating.*

Table 11. Linguistic terms and their corresponding TrNN for rating alternatives.

Linguistic Terms	Trapezoidal Neutrosophic Numbers (TrNN)
Low priority (LP)	$\{(0.0, 0.1, 0.2, 0.3); 0.75, 0.20, 0.20\}$
Below Priority (BP)	$\{(0.2, 0.3, 0.4, 0.5); 0.80, 0.20, 0.15\}$
Medium priority (MP)	$\{(0.4, 0.5, 0.6, 0.7); 0.85, 0.15, 0.15\}$
Very priority (VP)	$\{(0.6, 0.7, 0.8, 0.9); 0.90, 0.10, 0.15\}$
Extremely priority (EP)	$\{(0.7, 0.8, 0.9, 1.0); 0.95, 0.15, 0.00\}$

Table 12. Fixed data for Population density $(\tilde{m}_1)$, Rate of Literacy $(\tilde{m}_3)$ & Sex Ratio $(\tilde{m}_{10})$.

Alternative	Population Density $(\tilde{m}_1)$	Rate of Literacy $(\tilde{m}_3)$	Sex Ratio $(\tilde{m}_{10})$
Kolkata (L_{A1})	24306	86.31%	908
Howrah (L_{A2})	3306	83.31%	939
Berhampore (L_{A3})	1334	66.59%	958
Siliguri (L_{A4})	586	79.56%	970
Midnapore (L_{A5})	631	78.00%	966
Durgapur (L_{A6})	1099	76.21%	945

Table 13. Conversion table between fixed data to linguistic terms.

Linguistic Terms	Population Density $(\tilde{m}_1)$	Rate of Literacy $(\tilde{m}_3)$	Sex Ratio $(\tilde{m}_{10})$
Low priority (LP)	$\tilde{m}_1 < 500$	$\tilde{m}_3 < 65$	$\tilde{m}_{10} < 910$
Below priority (BP)	$500 \leq \tilde{m}_1 < 1000$	$65 \leq \tilde{m}_3 < 70$	$910 \leq \tilde{m}_{10} < 935$
Medium priority (MP)	$1000 \leq \tilde{m}_1 < 3000$	$70 \leq \tilde{m}_3 < 75$	$935 \leq \tilde{m}_{10} < 960$
Very priority (VP)	$3000 \leq \tilde{m}_1 < 15000$	$75 \leq \tilde{m}_3 < 80$	$960 \leq \tilde{m}_{10} < 985$
Extremely priority (EP)	$15000 \leq \tilde{m}_1$	$80 \leq \tilde{m}_3$	$985 \leq \tilde{m}_{10}$

Table 14. Comparison matrix in linguistic term between criterion's by the three DMs.

	Criteria	$\tilde{m}_1$	$\tilde{m}_2$	$\tilde{m}_3$	$\tilde{m}_4$	$\tilde{m}_5$	$\tilde{m}_6$	$\tilde{m}_7$	$\tilde{m}_8$	$\tilde{m}_9$	$\tilde{m}_{10}$
Decision Maker 1	Population density $(\tilde{m}_1)$	EI	VSNI	VSNI	SNI	EI	SNI	ANI	ANI	MNI	AI
	Investment Costs $(\tilde{m}_2)$	VSI	EI	VSNI	SNI	SI	VSNI	SNI	ANI	EI	ANI
	Rate of Literacy $(\tilde{m}_3)$	VSI	VSI	EI	EI	AI	EI	MI	EI	SI	SI
	Number of graduates $(\tilde{m}_4)$	SI	SI	EI	EI	SI	EI	EI	SNI	EI	SI
	Yearly income per person $(\tilde{m}_5)$	EI	SNI	ANI	SNI	EI	ANI	VSNI	ANI	SI	ANI
	Accessibility $(\tilde{m}_6)$	SI	VSI	EI	EI	AI	EI	SI	EI	SI	ANI
	Public Services $(\tilde{m}_7)$	AI	SI	MNI	EI	VSI	SNI	EI	SNI	EI	ANI
	Safety $(\tilde{m}_8)$	AI	AI	EI	SI	AI	EI	SI	EI	AI	AI
	Proximity of E. Institutes $(\tilde{m}_9)$	MI	EI	SNI	EI	SNI	SNI	EI	ANI	EI	MNI
	Sex Ratio $(\tilde{m}_{10})$	ANI	AI	SNI	SNI	AI	AI	AI	ANI	MI	EI
	Criteria	$\tilde{m}_1$	$\tilde{m}_2$	$\tilde{m}_3$	$\tilde{m}_4$	$\tilde{m}_5$	$\tilde{m}_6$	$\tilde{m}_7$	$\tilde{m}_8$	$\tilde{m}_9$	$\tilde{m}_{10}$
Decision Maker 2	Population density $(\tilde{m}_1)$	EI	SNI	ANI	VSNI	MNI	SNI	SNI	ANI	MNI	AI
	Investment Costs $(\tilde{m}_2)$	SI	EI	ANI	SNI	VSI	SNI	SNI	VSNI	MNI	VSNI
	Rate of Literacy $(\tilde{m}_3)$	AI	AI	EI	MI	VSI	EI	SI	EI	EI	VSI
	Number of graduates $(\tilde{m}_4)$	VSI	SI	MNI	EI	AI	MI	SI	ANI	MNI	SI
	Yearly income per person $(\tilde{m}_5)$	MI	VSNI	VSNI	ANI	EI	ANI	VSNI	ANI	MI	VSNI
	Accessibility $(\tilde{m}_6)$	SI	SI	EI	MNI	AI	EI	EI	EI	SI	ANI
	Public Services $(\tilde{m}_7)$	SI	SI	SNI	SNI	VSI	EI	EI	SNI	EI	VSNI
	Safety $(\tilde{m}_8)$	AI	VSI	EI	AI	AI	EI	SI	EI	VSI	VSI
	Proximity of E. Institutes $(\tilde{m}_9)$	MI	MI	EI	MI	MNI	SNI	EI	VSNI	EI	EI
	Sex Ratio $(\tilde{m}_{10})$	ANI	VSI	VSNI	SNI	VSI	AI	VSI	VSNI	EI	EI

Table 14. *Cont.*

	Criteria	$\tilde{m_1}$	$\tilde{m_2}$	$\tilde{m_3}$	$\tilde{m_4}$	$\tilde{m_5}$	$\tilde{m_6}$	$\tilde{m_7}$	$\tilde{m_8}$	$\tilde{m_9}$	$\tilde{m_{10}}$
Decision Maker 3	Population density ($\tilde{m_1}$)	EI	MNI	MNI	SNI	MI	ANI	ANI	VSNI	EI	VSI
	Investment Costs ($\tilde{m_2}$)	MI	EI	VSNI	ANI	SNI	MI	VSNI	ANI	SI	ANI
	Rate of Literacy ($\tilde{m_3}$)	MI	VSI	EI	MI	AI	SI	EI	EI	MI	MI
	Number of graduates ($\tilde{m_4}$)	SI	AI	MNI	EI	AI	SI	EI	EI	SI	VSI
	Yearly income per person ($\tilde{m_5}$)	MNI	SI	ANI	ANI	EI	VSNI	SNI	SNI	SNI	VSNI
	Accessibility ($\tilde{m_6}$)	AI	MNI	SNI	SNI	VSI	EI	MI	SNI	MI	VSNI
	Public Services ($\tilde{m_7}$)	AI	VSI	EI	EI	SI	MNI	EI	EI	SNI	ANI
	Safety ($\tilde{m_8}$)	VSI	AI	EI	EI	SI	SI	EI	EI	MNI	AI
	Proximity of E. Institutes ($\tilde{m_9}$)	EI	SNI	MNI	SNI	SI	MNI	SI	MNI	EI	MNI
	Sex Ratio ($\tilde{m_{10}}$)	VSNI	AI	MNI	VSNI	VSI	VSI	AI	ANI	MI	EI

Remark 5. *All data about Population density* ($\tilde{m_1}$) *& Sex Ratio* ($\tilde{m_{10}}$) *are collect from Census2011 [129] and Rate of Literacy* ($\tilde{m_3}$) *data is from Wikipedia [130] shown in Table 12. Those data are taken from authorised sources in the year 2011. Transformation of fixed data to linguistic term are shown in Table 13.*

Table 15. Comparison matrix in linguistic terms between criterion's and alternatives by three DMs.

	Criteria	$\tilde{m_1}$	$\tilde{m_2}$	$\tilde{m_3}$	$\tilde{m_4}$	$\tilde{m_5}$	$\tilde{m_6}$	$\tilde{m_7}$	$\tilde{m_8}$	$\tilde{m_9}$	$\tilde{m_{10}}$
Decision Maker 1	Kolkata (L_{A1})	EP	EP	EP	MP	MP	EP	EP	MP	EP	LP
	Howrah (L_{A2})	VP	MP	EP	LP	BP	VP	EP	LP	BP	MP
	Berhampore (L_{A3})	MP	BP	BP	LP	BP	BP	LP	LP	LP	MP
	Siliguri (L_{A4})	BP	MP	VP	LP	BP	MP	BP	BP	LP	VP
	Midnapore (L_{A5})	BP	BP	VP	MP	BP	MP	MP	MP	MP	VP
	Durgapur (L_{A6})	MP	MP	VP	MP	MP	BP	BP	MP	BP	MP
	Criteria	$\tilde{m_1}$	$\tilde{m_2}$	$\tilde{m_3}$	$\tilde{m_4}$	$\tilde{m_5}$	$\tilde{m_6}$	$\tilde{m_7}$	$\tilde{m_8}$	$\tilde{m_9}$	$\tilde{m_{10}}$
Decision Maker 2	Kolkata (L_{A1})	EP	EP	EP	MP	VP	EP	EP	VP	VP	LP
	Howrah (L_{A2})	VP	VP	EP	BP	MP	EP	VP	BP	MP	MP
	Berhampore (L_{A3})	MP	BP	BP	LP	LP	MP	LP	MP	LP	MP
	Siliguri (L_{A4})	BP	BP	VP	LP	MP	VP	MP	BP	LP	VP
	Midnapore (L_{A5})	BP	MP	VP	EP	MP	VP	BP	VP	MP	VP
	Durgapur (L_{A6})	MP	BP	VP	BP	BP	MP	BP	BP	BP	MP
	Criteria	$\tilde{m_1}$	$\tilde{m_2}$	$\tilde{m_3}$	$\tilde{m_4}$	$\tilde{m_5}$	$\tilde{m_6}$	$\tilde{m_7}$	$\tilde{m_8}$	$\tilde{m_9}$	$\tilde{m_{10}}$
Decision Maker 3	Kolkata (L_{A1})	EP	VP	EP	VP	VP	VP	EP	VP	EP	LP
	Howrah (L_{A2})	VP	VP	EP	MP	MP	VP	EP	BP	BP	MP
	Berhampore (L_{A3})	MP	BP	BP	BP	LP	BP	LP	BP	BP	MP
	Siliguri (L_{A4})	BP	MP	VP	LP	BP	MP	BP	MP	BP	VP
	Midnapore (L_{A5})	BP	MP	VP	VP	MP	BP	MP	VP	MP	VP
	Durgapur (L_{A6})	MP	BP	VP	BP	BP	BP	LP	MP	LP	MP

8. Numerical Illustration

The numerical computation using the said strategy mentioned in Section 4 for the proposed model mentioned in Section 5 associated with the data set mentioned in Section 6 is performed in this section. The following steps are followed by us for numerical computation. The flowchart of the selection process is also shown in Figure 3.

8.1. Step 1

Using the data of Table 14 associated with the Table 9 apply the NAHP formulas, which are described in Section 4.1. The consistency of the decision matrix is examined. The decision matrix is found out to be consistent (<0.1). The criterion's weight in crisp value is described in Table 16.

Table 16. Normalized criterion's weight using neutrosophic AHP.

Criteria	$\tilde{m}_1$	$\tilde{m}_2$	$\tilde{m}_3$	$\tilde{m}_4$	$\tilde{m}_5$	$\tilde{m}_6$	$\tilde{m}_7$	$\tilde{m}_8$	$\tilde{m}_9$	$\tilde{m}_{10}$
Criteria weight	0.0696	0.0677	0.1400	0.1197	0.0578	0.1074	0.0989	0.1478	0.0932	0.0979

Remark 6. *The crisp weight of all criteria is shown in Table 16 using NAHP. The values reflect that the criteria 'Safety' ($\tilde{m}_8$) is the most important followed by 'Rate of Literacy' ($\tilde{m}_3$), 'Number of Graduates' ($\tilde{m}_4$), 'Accessibility' ($\tilde{m}_6$), 'Public Services' ($\tilde{m}_7$), 'Sex ratio' ($\tilde{m}_{10}$), 'Proximity of E. Institutes' ($\tilde{m}_9$), 'Population Density' ($\tilde{m}_1$), 'Investment Costs' ($\tilde{m}_2$) and 'Yearly income per Person' ($\tilde{m}_5$) is the least significant criteria amongst the site selection criteria.*

8.2. Step 2

Utilization of the DMs data given in Table 14 with the linguistic term to converted TrNN using Table 9 we apply the neutrosophic weight calculating formulas, described in Section 4.2. The neutrosophic weight of all criteria is shown in Table 17 where $\beta_1, \beta_2, \beta_3$ & β_4 are the first, second, third, and forth entries of the trapezoidal neutrosophic numbers respectively and t for true, i for indeterminacy and f for false membership value.

Table 17. Depiction of neutrosophic criteria weight for criterion's.

Criteria	β_1	β_2	β_3	β_4	t	i	f
Population density ($\tilde{m}_1$)	0.0000	0.0536	0.0929	0.6438	0.7500	0.3000	0.2000
Investment costs ($\tilde{m}_2$)	0.0000	0.0528	0.0925	0.6458	0.7500	0.3000	0.2000
Rate of literacy ($\tilde{m}_3$)	0.0634	0.1146	0.1647	0.9274	0.8500	0.2000	0.1500
Number of graduates ($\tilde{m}_4$)	0.0000	0.0972	0.1454	0.9104	0.7500	0.3000	0.2000
Yearly income per person ($\tilde{m}_5$)	0.0000	0.0425	0.0791	0.6024	0.7500	0.3000	0.2000
Accessibility ($\tilde{m}_6$)	0.0000	0.0873	0.1340	0.8142	0.7500	0.3000	0.2000
Public services ($\tilde{m}_7$)	0.0000	0.0798	0.1247	0.7703	0.7500	0.3000	0.2000
Safety ($\tilde{m}_8$)	0.0649	0.1228	0.1734	0.9633	0.8000	0.2000	0.1500
Proximity of E. institutes ($\tilde{m}_9$)	0.0000	0.0751	0.1178	0.7849	0.7500	0.3000	0.2000
Sex ratio ($\tilde{m}_{10}$)	0.0000	0.0764	0.1222	0.7359	0.7500	0.3000	0.2000

8.3. Step 3

Application of NTOPSIS ranking model which is described in Section 4.3. The positive ideal solution (TP_j^+), negative ideal solution (TN_j^-), and relative closeness R_j with ranking on the basis of R_j values are represented in Table 18.

Table 18. Alternatives ranking with their adjacent data by using the **NTOPSIS** method.

Alternatives	TP_j^+	TN_j^-	$R_j = \frac{TN_j^-}{TP_j^+ + TN_j^-}$	**Ranking**
Kolkata (L_{A1})	0.1382	0.8824	0.8646	**1**
Howrah (L_{A2})	0.3527	0.6680	0.6545	**2**
Berhampore (L_{A3})	0.8321	0.1886	0.1847	**6**
Siliguri (L_{A4})	0.6078	0.4160	0.4063	**5**
Midnapore (L_{A5})	0.3555	0.6656	0.6518	**3**
Durgapur (L_{A6})	0.6058	0.4178	0.4082	**4**

8.4. Step 4

Using all data as the previous Section 8.3, we perform an analysis using by NCOPRAS method for ranking, which is discussed in Section 4.4. The De-Neutrosophication sum of beneficiary criteria (BC) is denoted by S_i^+, and the De-Neutrosophication sum of non-beneficiary criteria (NBC) is denoted by S_i^-. The factor Q_i is mentioned in Equation (39), and its percentage is denoted by R_i. All the computation values and ranking of the alternative on ascending order of R_i values are described in Table 19.

Table 19. Alternatives ranking and their adjacent data by using the **NCOPRAS** method.

Alternatives	S_i^+	S_i^-	Q_i	$R_i(\%)$	Ranking
Kolkata (L_{A1})	1.807561	0.056614	2.062153	100.0	**1**
Howrah (L_{A2})	1.58208	0.082565	1.756652	85.18	**2**
Berhampore (L_{A3})	1.099746	0.165103	1.187046	57.56	**6**
Siliguri (L_{A4})	1.356608	0.159694	1.446866	70.16	**5**
Midnapore (L_{A5})	1.653031	0.159694	1.743288	84.53	**3**
Durgapur (L_{A6})	1.361117	0.162181	1.44999	70.31	**4**

Remark 7. *From Table 18, Table 19, and Figure 6, we see that the ranking for alternative gives the same result for two methods NTOPSIS and NCOPRAS, respectively. So the decision maker can easily take the preferable sites for the mentioned alternatives. The sites 'Kolkata' come to the first position, Howrah becomes the second position, Midnapore becomes the third position, Durgapur becomes the fourth position, Siliguri becomes the fifth position, and Berhampore becomes the sixth position.*

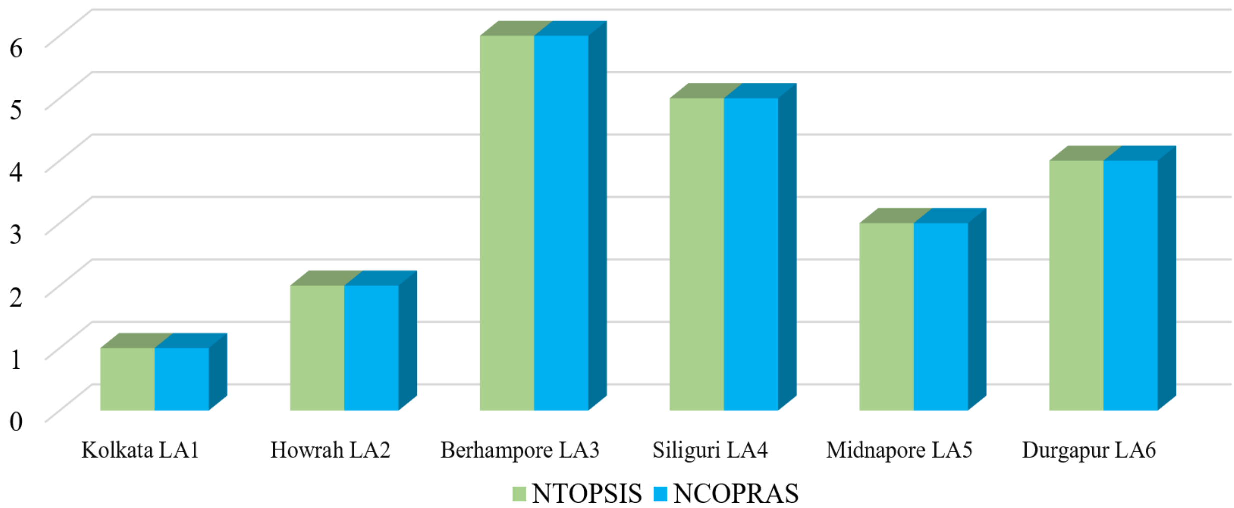

Figure 6. Comparative ranking diagram between NTOPSIS & NCOPRAS methods.

8.5. Computational Complexity

The computational complexity of the proposed fuzzy MCDM methodology has been described in this section. For example, the concept of computational complexity is also described in [131–133]. The number of mathematical operations has been performed to get the result is determined by time complexity which is denoted by TC in this study. We also assume l as the number of factors, k as the number of alternatives with N as the number of decision-makers. Thereafter, the following steps are taken to compute the computational complexity.

1. For NAHP, the comparison matrix is of l^2 entries; therefore, by N DMs gives $N \times l^2$ entries. Finding a comparison matrix has been Nl^2 operations needed. Then De-Neutrosophic process l^2 number of operations performed and to get normalized

De-Neutrosophic comparison matrix l^2 operations conducted. Thereafter to find the nth root and factor weight needed $2l + 1$ operations. Also, factor weight needs l^2 operations. Then factor sum and sum/weight are calculated by $2l$ operations. Finally, the consistency ratio is calculated by 3 more operations. Total operations conducted for NAHP are $N \times l^2 + l^2 + l^2 + 2l + 1 + l^2 + 2l + 3 = (N + 3)l^2 + 4(l + 1)$.

2. For Trapezoidal Neutrosophic Number (TrNN) factor weight calculated on comparison matrix with given N DMs total $N \times l^2$ entries. Calculation of geometric mean by $7l$ operations. The sum and inverse operations performed by 2 number of operations. Finally, factor weight calculation require $7l$ operations. Therefore, total calculations performed are $7l + 2 + 7l = 14l + 2$.
3. For NTOPSIS method, decision matrix is $k \times l$ entries therefore N DMs gives Nkl entries. The decision matrix has Nkl operations needed. Then normalized and weighted normalized decision matrix was calculated using $2kl + l$ operations. Finding positive and negative ideal solution there are $2l$ operations. To measure the relative closeness from the positive and negative ideal solutions there are $2kl$ operations performed, and the calculated total sum is by $2k$ number of operations. Finally, the comparison ratio and ranking of the alternatives need $2k$ number of operations. The total number of calculations conducted is $Nkl + 2kl + l + 2l + 2kl + 2k + 2k = (N + 4)kl + 4k + 3l$.
4. For NCOPRAS techniques, up to weighted normalized decision matrix $Nkl + 2kl + l$ operations are performed, which is already done in NTOPSIS. Then calculated sum of beneficial and non-beneficial attributes $2k$ number of operations needed. For the De-Neutrosophic method, $2k$ operations were performed. Then find Q_i values for k number of operations needed and lastly, k operations performed to rank the alternatives. The total calculation performed are $Nkl + kl + l + 2k + 2k + k + k = (N + 1)kl + 6k + l$.

Time complexity of this study TC is calculated as factor $l = 10$, alternatives $k = 6$ and decision maker $N = 3$ for the present problem as follows

- For NAHP, number of calculations are $(3 + 3) \times 10^2 + 4 \times 10 + 4 = 644$.
- For weight, number of operations are $14 \times 10 + 2 = 142$.
- For FTOPSIS, number of operations are $(3 + 4) \times 6 \times 10 + 4 \times 6 + 3 \times 10 = 474$.
- For FCOPRAS, number of calculations are $(3 + 1) \times 6 \times 10 + 6 \times 6 + 10 = 286$.

Then the time complexity $TC = 644 + 142 + 474 + 286 = 1546$.

9. Sensitivity Analysis

Sensitivity analysis generally expresses the different ranking of the alternatives in a different environment. As it is known that decision-making depends on various conflicting criteria, in sensitivity analysis, removal of criteria or interchange of criterion's weight with respect to some conditions can be executed. Thus, in this study, three different cases have been considered. Different rankings obtained under these cases using two MCDM tools NTOPSIS and NCOPRAS are represented graphically, and a detailed explanation is given as to why these cases are taken.

9.1. Removing Investment Cost ($\tilde{m}_2$)

In the first case, the removal of the criteria and investment cost has been considered. Several times, it is seen that during the construction of a University, a government or charitable trust offers funds for the construction. Thus, in this scenario, the investor need not necessarily think about the investment cost.

Remark 8. *Ranking obtained under removal of investment cost shows Table 20 and Figure 7 that the alternatives 'Kolkata', 'Berhampore' and 'Siliguri' remained consistent with position first, sixth and fifth, respectively.*

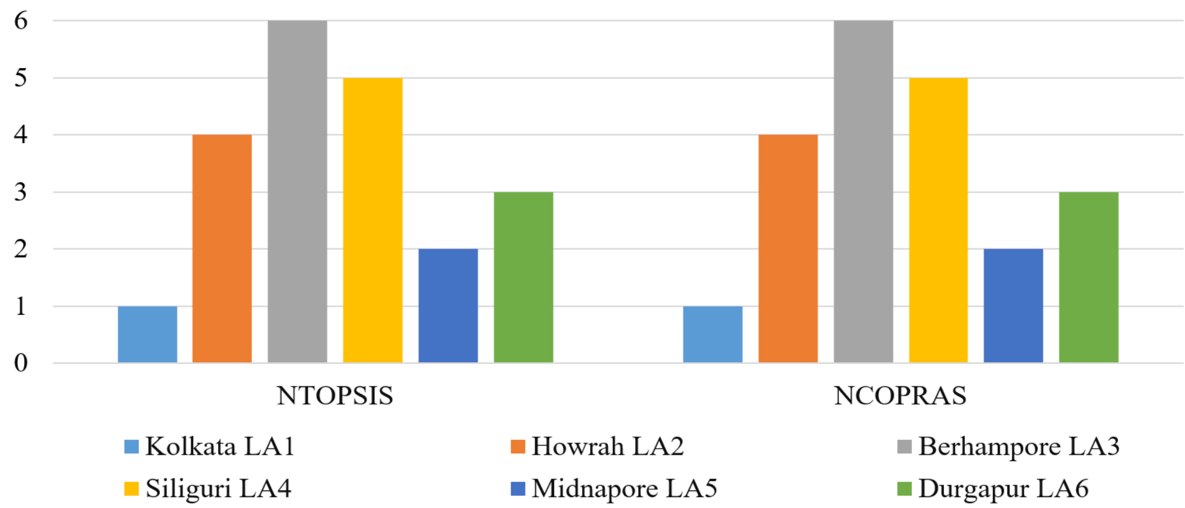

Figure 7. Depiction of ranking alternatives by removing Investment Cost $(\tilde{m}_2)$.

Table 20. Ranking alternatives by removing Investment Cost $(\tilde{m}_2)$.

Alternatives	Ranking Using NTOPSIS	Ranking Using NCOPRAS
Kolkata (L_{A1})	1	1
Howrah (L_{A2})	4	4
Berhampore (L_{A3})	6	6
Siliguri (L_{A4})	5	5
Midnapore (L_{A5})	2	2
Durgapur (L_{A6})	3	3

Note: Here we make a note of how this case numerical is done. Removing the criteria of Investment Cost $(\tilde{m}_2)$ from the Table 15 and TrNN weight from Table 17 in Section 8.2, we get a weighted normalized decision matrix where the criteria weight of Investment Cost $(\tilde{m}_2)$ may not zero or if we forcefully take zero, then all criteria weight are not normalized. So we need to calculate TrNN criteria weight for removing Investment Cost $(\tilde{m}_2)$ using the formula in Section 4.2 and then calculate the weighted normalized matrix and apply NTOPSIS and NCOPRAS as described in Sections 4.3 and 4.4 respectively.

9.2. Removing Accessibility $(\tilde{m}_6)$

The criteria 'accessibility' has been removed in the second case considering the three environments:

1. Government has proposed public accessibility.
2. The investors might set up its own accessibility.
3. Fully residential university need not require accessibility.

Remark 9. *Ranking obtained under this environment reveals in Table 21 and Figure 8 shows that the locations 'Kolkata', 'Durgapur', 'Siliguri' and 'Berhampore' are at the same position whereas the alternatives 'Howrah', and 'Midnapore' rankings have been interchanged.*

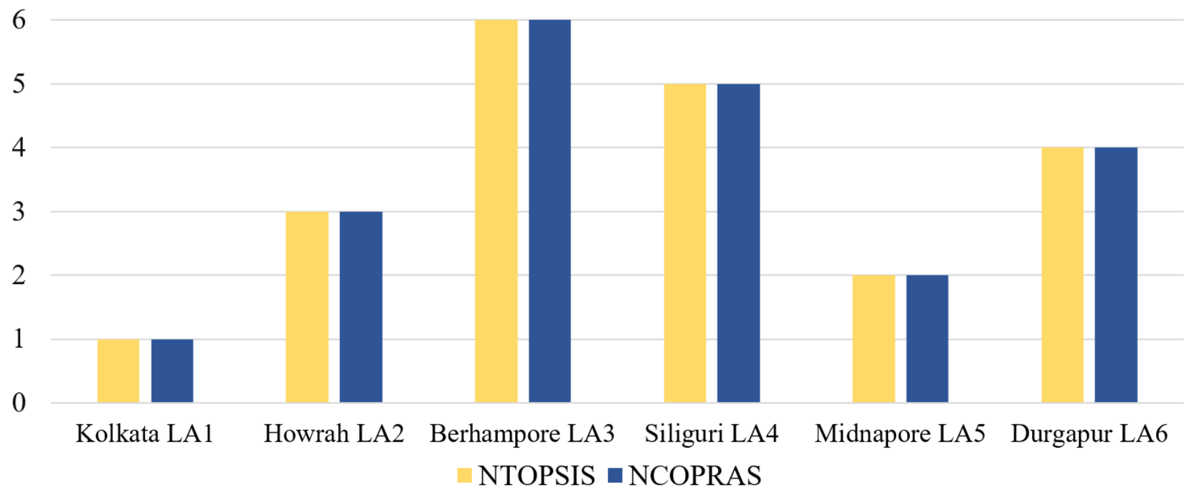

Figure 8. Pictorial of alternatives ranking by removing Accessibility $(\tilde{m}_6)$.

Table 21. Alternatives ranking by removing criteria Accessibility $(\tilde{m}_6)$.

Alternatives	Ranking Using NTOPSIS	Ranking Using NCOPRAS
Kolkata (L_{A1})	1	1
Howrah (L_{A2})	3	3
Berhampore (L_{A3})	6	6
Siliguri (L_{A4})	5	5
Midnapore (L_{A5})	2	2
Durgapur (L_{A6})	4	4

Note: For calculation purposes, if we remove the criteria Accessibility $(\tilde{m}_6)$ from the Table 15 and TrNN weight from Table 17 in Section 8.2 we get weighted normalized decision matrix where the criteria weight of Accessibility $(\tilde{m}_6)$ may not be zero or if we forcefully take zero then all criteria weight are not normalized. So we calculated TrNN criteria weight for removing Accessibility $(\tilde{m}_6)$ as previous Section 9.1 and ranking the alternatives using the NTOPSIS and NCOPRAS methods as described in Sections 4.3 and 4.4 respectively.

9.3. Removing Proximity of Educational Institute $(\tilde{m}_9)$

Individual Universities has it's own infrastructure, such as libraries, research cell, academic development cell, etc. Thus, the proximity of educational institutes to the University may not be considered necessary.

Remark 10. *Ranking obtained under this condition represents in Table 22 and Figure 9 that the same ranking of Section 9.2 order for the locations 'Kolkata', 'Durgapur', 'Siliguri' and 'Berhampore'. It is observed that consistent ranking is obtained for the two cases taken in sensitivity analysis, i.e., 'removal of accessibility* $(\tilde{m}_6)$*' and 'removal of the proximity of educational institute* $(\tilde{m}_9)$*'.*

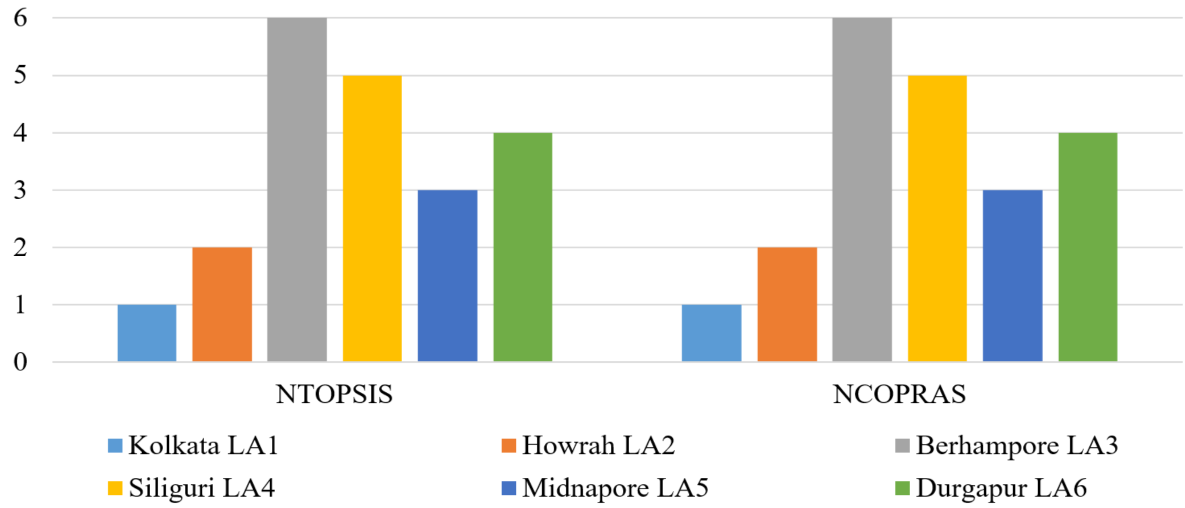

Figure 9. Depiction of alternatives ranking by removing Proximity of Educational institute $(\tilde{m}_9)$.

Table 22. Alternatives ranking by removing Proximity of Educational institute $(\tilde{m}_9)$.

Alternatives	Ranking Using NTOPSIS	Ranking Using NCOPRAS
Kolkata (L_{A1})	1	1
Howrah (L_{A2})	2	2
Berhampore (L_{A3})	6	6
Siliguri (L_{A4})	5	5
Midnapore (L_{A5})	3	3
Durgapur (L_{A6})	4	4

Note: For calculation purposes, if we remove the criteria Proximity of Educational institute $(\tilde{m}_9)$ from the Table 15 and TrNN weight from Table 17 in Section 8.2 we get new weighted normalized decision matrix where the TrNN criteria weight of Proximity of Educational institute $(\tilde{m}_9)$ may not be zero or if we forcefully take zero then all criteria weight are not normalized. So we calculated TrNN criteria weight for removing Proximity of Educational institute $(\tilde{m}_9)$ as above Section 9.1 and ranking the alternatives on MCDM method NTOPSIS and NCOPRAS as described in Sections 4.3 and 4.4 respectively.

10. Conclusions and Future Research Scope

The neutrosophic MCDM method uses a screening methodology to find the solution for different complex problems with uncertain data. It is capable of finding valuable information for the decision-makers by comparing a host of different parameters before making the final conclusion. Finding the best location for setting up a women's university is a major social and economic concern requiring the trade-off and weighting of various factors. The core aim of this study is the selection of the best location for women's universities by considering social and national needs. This study provides a model to the stakeholder, e.g., investors and government, who are searching for an optimal site keeping in mind requirements like Universities site selection, hospital site selection, landfill site selection, college site selection, etc.

In this work, two different selection methods have been used for choosing an optimum site. The most commonly used methods that have been elaborated upon include AHP, TOPSIS, COPRAS, and neutrosophic set theory. A new de-neutrosophication technique has been introduced and applied in this present work. Comparisons between the two methods NTOPSIS and NCOPRAS have been carried out. This comparison showed that the proposed methodology is reliable. Sensitivity analysis has been carried out by incorporating different possible weights and using a combination of influencing factors to accommodate different organizational needs.

The following steps are taken for doing the numerical study:

1. Check the consistency of the decision matrix using Neutrosophic AHP.
2. Obtain the neutrosophic weight of the criteria to evaluate the weighted decision matrix.
3. Calculate the nearest distance from the positive ideal solution, and the farthest distance from the negative ideal solution is calculated using the MCDM method Neutrosophic TOPSIS.
4. Determine the maximizing and minimizing index values, and consequently, the attributes of maximizing and minimizing indexes are the assessment of the results examined individually using the Neutrosophic COPRAS method.

Since the paper deals with optimal site selection for women's universities, we considered alternative locations for numerical study. We consulted experts and conducted a thorough literature review to ascertain relevant criteria. We considered ten important criterion's for the problem. This methodology can be extended and applied in numerous fields. Some of the future scope/extensions are

- the methodology can be used for setting up a private university, fully research-oriented institute, etc.,
- different sub-criterion may be taken for each criterion,
- different de-neutrosophication techniques associated with different efficient MCDM methods, like MIVES, WASPAS, CoCOSo, PROMETHEE, VIKOR may be applied,
- different uncertain environments may be considered like hesitant neutrosophic environment, Pythagorean fuzzy,
- same methodology may be extended with more alternatives,
- different new distance measures may be used.

The findings of the work presented here may be helpful for a decision-maker who deals with the site selection problem with some uncertainties in data. Our results should also encourage a more straightforward method with appropriate decision outcomes.

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A new synergistic strategy for ranking restaurant locations: A decision-making approach based on the hexagonal fuzzy numbers

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Description: This research addresses the problem of restaurant locations ranking with applications for a cosmopolitan big city like Kolkata, India. A restaurant selection is based on occasions, spending capability, environment, location, comfort, quality of the food etc. In this research paper an exhaustive set of factors and sub-factors is taken into consideration to select and rank restaurants situated at different locations in the city of Kolkata with a population of around fifteen million. The ranking of restaurants depends on complex, conflicting qualitative attributes. In the paper hexagonal fuzzy numbers (HFN) have been used to suitably depict the imprecise uncertain environment. HFN, its distance measure and defuzzification have been applied to deal with the hesitancy and impreciseness of the decision makers. Analytic hierarchy process (AHP) has been used as a Multi Criteria Decision Making (MCDM) tool to obtain factors and …

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A NEW SYNERGISTIC STRATEGY FOR RANKING RESTAURANT LOCATIONS: A DECISION-MAKING APPROACH BASED ON THE HEXAGONAL FUZZY NUMBERS

Kamal Hossain Gazi[1,*], Sankar Prasad Mondal[1], Banashree Chatterjee[2], Neha Ghorui[3], Arijit Ghosh[4] and Debashis De[5]

Abstract. This research addresses the problem of restaurant locations ranking with applications for a cosmopolitan big city like Kolkata, India. A restaurant selection is based on occasions, spending capability, environment, location, comfort, quality of the food etc. In this research paper an exhaustive set of factors and sub-factors is taken into consideration to select and rank restaurants situated at different locations in the city of Kolkata with a population of around fifteen million. The ranking of restaurants depends on complex, conflicting qualitative attributes. In the paper hexagonal fuzzy numbers (HFN) have been used to suitably depict the imprecise uncertain environment. HFN, its distance measure and defuzzification have been applied to deal with the hesitancy and impreciseness of the decision makers. Analytic hierarchy process (AHP) has been used as a Multi Criteria Decision Making (MCDM) tool to obtain factors and sub-factors weights. TOPSIS and COPRAS methods were used for ranking different restaurant locations. Using comparative analysis it is shown that HFN with the TOPSIS and COPRAS method gives better result than other fuzzy numbers. The sensitivity analysis portion also gives a direction for taking a suitable decision in different possible scenario.

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1. Introduction

In today's fast paced life, restaurant not only serves food they are a source of pleasant memories. Competition of restaurant business is increasing as there are more dining alternative. The results identified the important factor of judging the authenticity of restaurants. Further it was found that localness has scored more than authenticity and the two of them were linked [1]. The demand of restaurants are increasing throughout the globe. Customer expectations from restaurants are increasing and they choose restaurant based on their pref-

Keywords. HFN, MCDM, FAHP, FTOPSIS, FCOPRAS, Defuzzification, Decision maker (DM), Restaurant selection.

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erences. This study used mean end approach to identify customer's choice among the three different segment of restaurants *i.e.* casual restaurants, fast food restaurants and fine dining restaurants [2]. Potential Customers search information for selecting restaurant based on the desired service they want to avail. Information search survey was conducted in the city of Zaragoza in the north-east of Spain. Further the motivating factors for these activities were determined [3]. MCDM tools Fuzzy AHP (FAHP) and Fuzzy TOPSIS (FTOPSIS) were applied to evaluate the factors weight and ranking of sites [4].

There are several factors and sub-factors that influences on preferring a location while choosing restaurant. Thus Multi Criteria Decision Making (MCDM) can be applied to solve such problem. MCDM is an important branch of Operation Research. It transforms real life complex decision making to a logical conclusion. MCDM incorporates numerous qualitative, contradicting and/or quantitative factors and sub-factors to a logical conclusion that needs a consensus [5]. MCDM is most extensively used decision methodologies in several areas such as business, energy, economy, environment, production, sustainability and so on [4,6]. MCDM techniques augment the standard of decision making. There are several studies [7–9] that have exhibited the vitality of this space.

In real world situations uncertainty, vagueness, indeterminacy are prevalent. Fuzzy concept is suitable to express such situations. In this paper Fuzzy MCDM methods are used. Many researchers have used different uncertain environment based on the suitability in real life problems. The authors name, application area, number of factors and sub-factors, uncertain environmental category and MCDM techniques are described in Table 1.

Kolkata, the city under our consideration has seven locations where a cluster of restaurants are present. For different location, the attributes and qualities differs but in a particular location restaurants are homogeneous. For example China town has a cluster of restaurants providing Chinese food of similar quality. An individual willing to avail restaurant food first need to decide which location he/she will travel to have access to a restaurant. This research helps in location selection and ranking of restaurant locations. This paper attempts at the following:

(1) Selection of different locations for restaurant selection in the city of Kolkata and ranking them on the basis of multiple conflicting factors and sub-factors taking into account the evaluation of multiple decision makers (DMs).
(2) Identification of different factors and sub-factors influencing location selection for restaurant. Developing a comparison matrix using hexagonal fuzzy numbers (HFN) suitable for analytic hierarchy process (AHP). Obtaining Fuzzy HFN weight for all the factors and sub-factors.
(3) Ranking of different locations using Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) and Fuzzy Complex Proportional Assessment (Fuzzy COPRAS or FCOPRAS).
(4) To carry out sensitivity analysis and comparative analysis for checking reliability and sensitivity of our model.

A list of factors and sub-factors used in literature have been depicted in Table 2.

Various researchers integrated fuzzy numbers with MCDM tools like AHP, TOPSIS, COPRAS. In this research, Hexagonal Fuzzy Numbers with MCDM approach has been used for ranking restaurant locations and a detailed sensitivity analysis has been carried out which is a novelty of this research. Figure 1 describes the successive steps followed in this study.

The paper is arranged in the following way: Section 2 briefly describes the concept of fuzzy set and fuzzy numbers, α-cut of fuzzy number, HFN and respective arithmetic operations. Distance measure, defuzzification formulae are also represented in this section. Section 3 describe the MCDM technique AHP, Hexagonal fuzzy weight, Fuzzy TOPSIS and Fuzzy COPRAS. Description of factors and sub-factors, alternatives and numerical application are covered in Sections 4 and 5 respectively. Section 6 represents the Numerical calculation of DM's data. Section 7 portray about Comparative analysis. Section 8 represents sensitivity analysis. Section 9 discusses the managerial insights. Finally, conclusion and future research scope are covered in Section 10.

TABLE 1. Literature on relevant Fuzzy MCDM problems showing number of factors and sub-factors, type environment and MCDM techniques used.

Authors of the article	Application area	Numbers of factor and sub-factors	Environment category	MCDM techniques
Ghorui *et al.* [4]	Shopping mall site selection problem	7 Factors and 17 sub-factors	Triangular fuzzy numbers (TFN)	Fuzzy AHP, Fuzzy TOPSIS
Sarkar *et al.* [6]	Selecting best family car	11 Factors	Triangular fuzzy numbers (TFN)	FTOPSIS, FMARCOS, FVIKOR
Biswas *et al.* [10]	Medical representative recruitment system	4 Factors	Single valued trapezoidal neutrosophic numbers	Fuzzy TOPSIS
Ghosh *et al.* [11]	Site selection of electric vehicle charging station	4 Factors and 13 sub-factors	Hexagonal fuzzy numbers (HNF)	FAHP, FTOPSIS, FCOPRAS
Biswas *et al.* [12]	Most suitable tablet selection	6 Factors	Single valued neutrosophic number	TOPSIS
Hezam *et al.* [13]	COVID-19 vaccination priority for different groups	4 Factors and 15 sub-factors	Triangular neutrosophic fuzzy set	AHP, TOPSIS
Tanoumand *et al.* [14]	Selecting cloud computing	6 Factors	Triangular fuzzy numbers (TFN)	FAHP
Ali *et al.* [15]	Measuring the possibility of cloud adoption for software tasting	10 Factors and 70 sub-factors	Triangular fuzzy numbers (TFN)	FMCDM
Stankovi *et al.* [16]	Road traffic risk analysis	6 Factors	Triangular fuzzy numbers (TFN)	Fuzzy MARCOS
Tzeng *et al.* [17]	Restaurant location in Taipei	5 Factors and 11 sub-factors	Linguistic variable in decimal numbers	AHP and VIKOR
Timor *et al.* [18]	Fast-food restaurant site selection	7 Factors and 36 sub-factors	Linguistic variable in decimal numbers	AHP
Karasan *et al.* [19]	Residential construction site selection	4 Factors and 14 sub-factors	Hesitant fuzzy numbers	CODAS
Moatya *et al.* [20]	A site selection decision making process	6 Factors and 26 sub-factors	Linguistic variable in decimal numbers	AHP and TOPSIS
Sriniketha *et al.* [21]	Plant location selection	4 Factors and 13 sub-factors	Linguistic variable in decimal numbers	AHP and PROMETHEE
Chatterjee *et al.* [22]	Hospital location selection	3 Factors and 11 sub-factors	Linguistic variable in decimal numbers	AHP and multi factor evaluation
Sun [23]	Site selection for EVCSs	4 Factors and 19 sub-factors	Linguistic variable in decimal numbers	AHP and TOPSIS
Ramu *et al.* [24]	Airport site selection	14 Factors	Fuzzy numbers	AHP and FAHP
Wibisono *et al.* [25]	Selection of cafe location	5 Factors and 16 sub-factors	Priority analysis through cluster matrix	AHP
Chen *et al.* [26]	Sustainable selection of a teahouse location	11 Factors	Molt-Karlo simulation method	WASPAS and EDAS
In this paper	Location selection for a restaurant	5 Factors and 17 sub-factors	Hexagonal fuzzy numbers (HFN)	FAHP, FTOPSIS and FCOPRAS

2. Preliminaries

2.1. Fuzzy set

Fuzzy sets are the set whose every element has a degree of membership value. The fuzzy concept was first introduced by Zadeh [29, 30]. There are several application of Fuzzy set theory in different domains like Differential equation [31, 32], linear programming problem [33–35], non-linear programming problem [36, 37], decision making problem [38, 63–66] etc.

TABLE 2. Factors and sub-factors for restaurant location selection, source: [4, 17, 18, 27, 28].

Factors	Sub-factors
Food	* Price of food * Food quality * Variety of food * Taxes * Site and development costs
Service	* Service quality * Behavior of staff * Quick service * Pleasant physical environment * Customer satisfaction * Staff members are friendly and helpful
Image	* Past experience * Word of mouth * Online review * Brand reputation * Sales promotion * Store size * Building's condition
Location	* Area * Parking Capacity * Safety/Crime Rates * Clean and comfortable * Noise and air pollution free * Convenience of garbage disposal * Residential areas * Industrial areas * Shopping center * Sports and cultural areas * Business areas * Educational areas * Distance to nearest highway
Occasion	* Quick meal/convenience * Social occasion * Business necessity * Celebration * Development of nearby areas * Future development

Definition 1 (Fuzzy set)**.** Let Φ be a set (finite or infinite). Let $\tilde{S}$ be a set contained in Φ. A function $\mu_{\tilde{S}} : \Phi \to [0,1]$ is called a membership function. If $x \in \Phi$ then $\mu_{\tilde{S}}(x)$ the degree of membership of x in Φ.

Definition 2 (α-cut of a fuzzy set)**.** The α-cut or α-level set of the fuzzy set $\tilde{S}$ of Φ is a classical set $\tilde{S}_\alpha$ which contains all members of Φ such that membership values of ϕ ($\in \tilde{S}$) bigger than or equal to α *i.e.* $\tilde{S}_\alpha = \{\phi : \mu_{\tilde{S}}(\phi) \geq \alpha, \phi \in \Phi\}$, $\alpha \in [0,1]$.

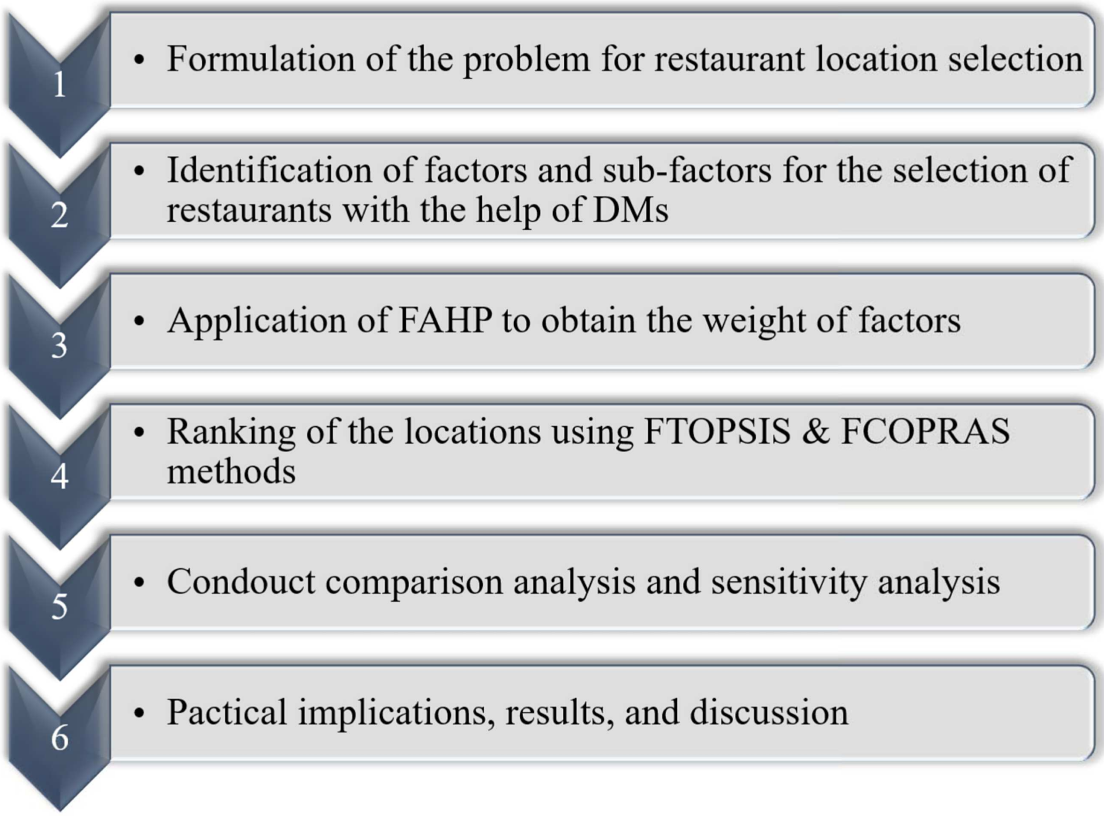

FIGURE 1. The structural framework of this paper.

Definition 3 (Strong α-cut of a fuzzy set)**.** The strong α-cut or strong α-level set of the fuzzy set $\tilde{S}$ of Φ is a classical set $\tilde{S}_{\tilde{\alpha}}$ which contains all members of Φ such that membership values of ϕ $(\in \tilde{S})$ bigger than α *i.e.* $\tilde{S}_{\tilde{\alpha}} = \{\phi : \mu_{\tilde{S}}(\phi) > \alpha, \phi \in \Phi\}$, $\alpha \in [0, 1]$.

2.2. Concept of fuzzy numbers:

An extension of real number set $\mathbb{R}$ with membership function is called fuzzy number [39], *i.e.* fuzzy numbers are connected set of possible values with the membership function $\tilde{\lambda}(\in [0, 1])$. The weight of the element is called membership value and the function through which the weight is assigned is called membership function.

Definition 4 (Normal fuzzy set [40])**.** A fuzzy set is called normal fuzzy set if its core (*i.e.*, the α-cut set along $\alpha = 1$) is non-empty.

Definition 5 (Convex fuzzy set [41])**.** A fuzzy set $\tilde{S}$ is said to be convex fuzzy set, if $\tilde{S}(\gamma x + (1-\gamma)y) \geq \min\{\tilde{S}(x), \tilde{S}(y)\}$ for $x, y \in \Phi$ and $\gamma \in (0, 1)$.

2.3. Hexagonal fuzzy number

There are many research papers published on fuzzy numbers, that developed and used fuzzy numbers [24]. Depending on the need of the problem, researchers can use triangular fuzzy numbers (TFN) [4, 6, 14–16], trapezoidal fuzzy numbers (TrFN) [42], pentagonal fuzzy numbers (PFN) [43], hexagonal fuzzy numbers (HFN) [44] and hesitant fuzzy numbers [19]. In this paper hexagonal fuzzy numbers have been used.

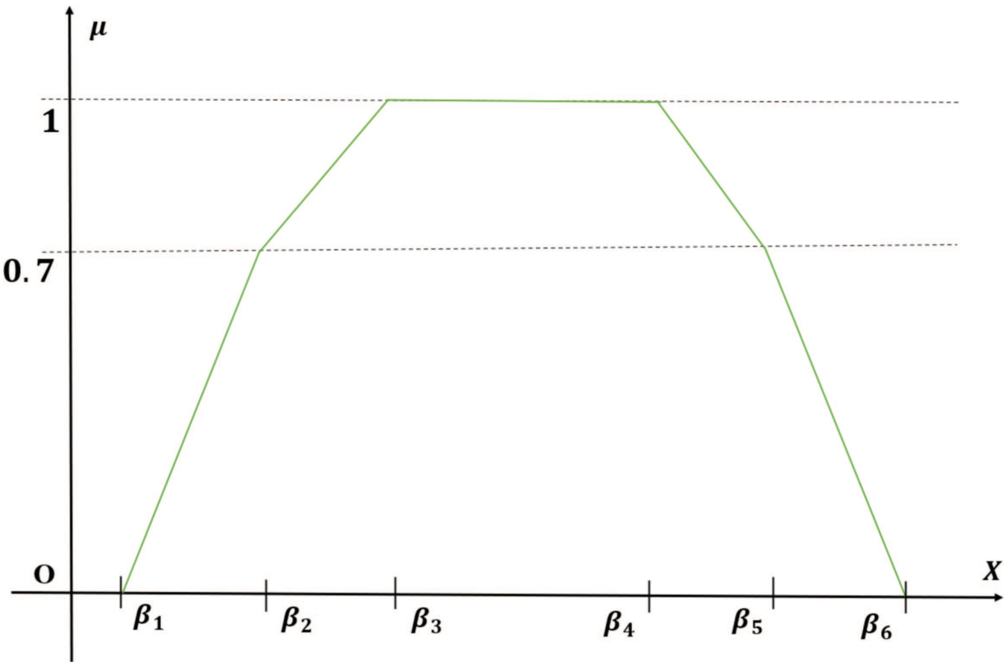

FIGURE 2. Geometric representation of hexagonal fuzzy number.

Definition 6 (Hexagonal fuzzy numbers (HFN) [44]). A fuzzy number $\tilde{H}(\phi) = \left\{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}(\phi)}\right\}$ is said to be hexagonal fuzzy number (HFN) where $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6$ are real numbers with ascending order, $0 \leq r, s \leq 1$ and its membership function $\mu_{\tilde{H}}(\phi)$ is defined as

$$\mu_{\tilde{H}}(\phi) = \begin{cases} 0 & \text{if } \phi \leq \beta_1 \\ r\frac{(\phi-\beta_1)}{(\beta_2-\beta_1)} & \text{if } \beta_1 \leq \phi \leq \beta_2 \\ r + (1-r)\frac{(\phi-\beta_2)}{(\beta_3-\beta_2)} & \text{if } \beta_2 \leq \phi \leq \beta_3 \\ 1 & \text{if } \beta_3 \leq \phi \leq \beta_4 \\ s + (1-s)\frac{(\beta_5-\phi)}{(\beta_5-\beta_4)} & \text{if } \beta_4 \leq \phi \leq \beta_5 \\ s\frac{(\beta_6-\phi)}{(\beta_6-\beta_5)} & \text{if } \beta_5 \leq \phi \leq \beta_6 \\ 0 & \text{if } \beta_6 \leq \phi. \end{cases} \tag{1}$$

Here, $\beta_1 \leq \beta_2 \leq \beta_3 \leq \beta_4 \leq \beta_5 \leq \beta_6$ with all β_i $(i = 1, 2, \ldots, 6)$ real constants and $0 < r, s < 1$.

Figure 2, represents a geometric representation on a particular hexagonal fuzzy number $\tilde{H}(\phi)$ where fuzzy set is

$$\tilde{H}(\phi) = \left\{(0.5, 1, 2, 4, 5.5, 6.5; 0.5, 0.6); \mu_{\tilde{H}(\phi)}\right\}.$$

Definition 7. A fuzzy number $\tilde{H} = \left\{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}(\phi)}\right\}$ is a hexagonal fuzzy number if it's membership function $\mu_{\tilde{H}}$ satisfies following conditions $\mu_{\tilde{H}} : \mathbb{R} \to I = [0, 1]$, and defined as

(1) $\mu_{\tilde{H}}$ is upper semi-continuous;
(2) There exists real numbers β_1, β_2, β_3, β_4, β_5 and β_6 such that $\beta_1 \leq \beta_2 \leq \beta_3 \leq \beta_4 \leq \beta_5 \leq \beta_6$ and
   (a) $\mu_{\tilde{H}}(\phi)$ is monotonically increasing on $[\beta_1, \beta_2]$ and $[\beta_2, \beta_3]$,

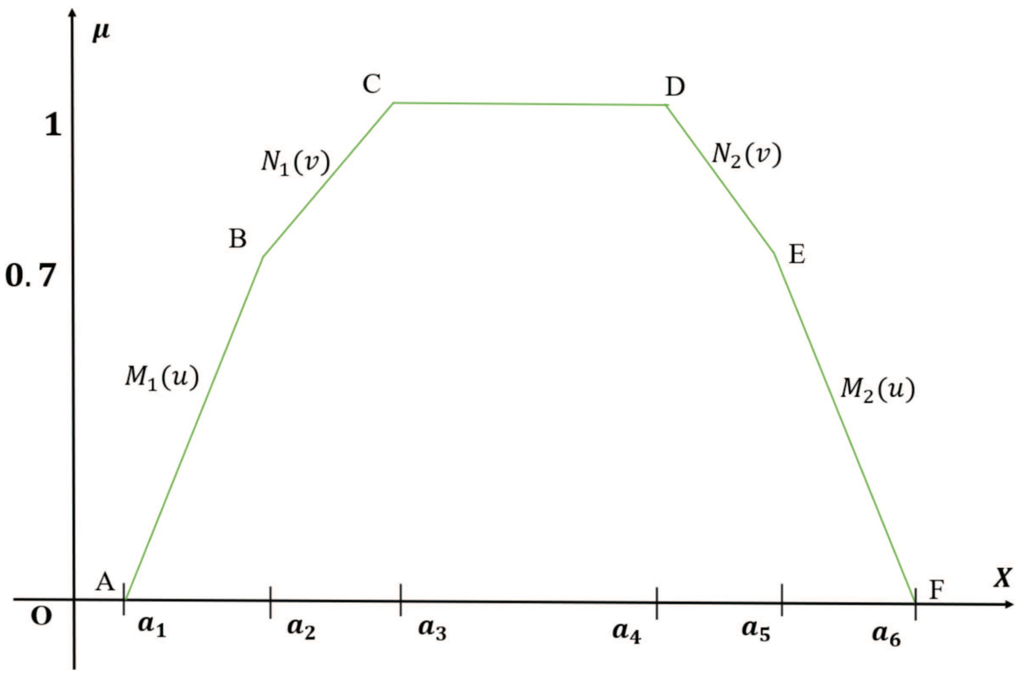

FIGURE 3. Geometric description of hexagonal fuzzy number $\tilde{H}$ where membership function $\mu_{\tilde{H}}(\phi)$ is bounded continuous and $\phi \in [0, 1]$.

(b) $\mu_{\tilde{H}}(\phi)$ is monotonically decreasing on $[\beta_4, \beta_5]$ and $[\beta_5, \beta_6]$,
(c) $\mu_{\tilde{H}}(\phi) = 1$ when $\beta_3 \leq \phi \leq \beta_4$,
(d) $\mu_{\tilde{H}}(\phi) = 0$, if ϕ lies outside the interval $[\beta_1, \beta_6]$.

2.4. α-cut of HFN

The alpha cut of $\tilde{H}$ on fuzzy set is denoted by $\tilde{H}_\alpha$ and constructed by the elements of $\tilde{H}$ whose membership value is not less than α. α-cut concept is briefly described in details [45] and [46].

Definition 8. Let $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6); \mu_{\tilde{H}}\}$ be a hexagonal fuzzy number and the membership function $\mu_{\tilde{H}}$, then α-cut set of $\tilde{H}$ is

$$\tilde{H}_\alpha = \{\phi \in \Phi | \mu_{\tilde{H}}(\phi) \geq \alpha\} = \begin{cases} [M_1(\alpha), M_2(\alpha)]; & \text{for } \alpha \in [0, 0.5] \\ [N_1(\alpha), N_2(\alpha)]; & \text{for } \alpha \in [0.5, 1] \end{cases} \tag{2}$$

Note 1. *Here, we consider membership function $\mu_{\tilde{H}}(\phi)$ are continuous function. Four functions of $\tilde{H}_\alpha$ are $M_1(u), N_1(v), N_2(v), M_2(u)$ satisfies the properties:*

(1) The non decreasing continuous bounded function $M_1(u)$ expand in $[0, 0.5]$.
(2) The non decreasing continuous bounded function $N_1(u)$ expand in $[0.5, 1]$.
(3) The non increasing continuous bounded function $N_2(u)$ expand in $[0.5, 1]$.
(4) The non increasing continuous bounded function $M_2(u)$ expand in $[0, 0.5]$.

A bounded continuous membership function $\mu_{\tilde{H}}(\phi)$ of HFN $\tilde{H}$ is graphically represented in Figure 3. Also the function $M_1(u), N_1(v), N_2(v), M_2(u)$ are described.

If we get α-cut operations on $\tilde{H}$ shall be obtained as follows for $\alpha \in [0,1]$.

Consider, $M_1(x) = r\frac{\phi-\beta_1}{\beta_2-\beta_1} = \alpha$

or,

$$\phi = \frac{\alpha}{r}(\beta_2 - \beta_1) + \beta_1. \tag{3}$$

Similarly, $M_2(x) = s\frac{\beta_6-\phi}{\beta_6-\beta_5} = \alpha$

or, $\beta_6 - \phi = \frac{\alpha}{s}(\beta_6 - \beta_5)$

or,

$$\phi = -\frac{\alpha}{s}(\beta_6 - \beta_5) + \beta_6. \tag{4}$$

This implies $[M_1(\alpha), M_2(\alpha] = \left[\frac{\alpha}{r}(\beta_2-\beta_1)+\beta_1, -\frac{\alpha}{s}(\beta_6-\beta_5)+\beta_6\right]$ when $\alpha \in [0, 0.5]$.

Again, $N_1(\phi) = r + (1-r)\frac{\phi-\beta_2}{\beta_3-\beta_2}$

or, $\phi - \beta_2 = \frac{\alpha-r}{1-r}(\beta_3 - \beta_2)$

or,

$$\phi = \frac{\alpha - r}{1 - r}(\beta_3 - \beta_2) + \beta_2. \tag{5}$$

Similarly, $N_2(\phi) = s + (1-s)\frac{\beta_5-\phi}{\beta_5-\beta_4} = \alpha$

or, $\beta_5 - \phi = \frac{\alpha-s}{1-s}(\beta_5 - \beta_4)$

or,

$$\phi = -\frac{\alpha - s}{1 - s}(\beta_5 - \beta_4) + \beta_5. \tag{6}$$

This implies $[N_1(\alpha), N_2(\alpha] = \left[\frac{\alpha-r}{1-r}(\beta_3-\beta_2)+\beta_2, -\frac{\alpha-s}{1-s}(\beta_5-\beta_4)+\beta_5\right]$ when $\alpha \in [0.5, 1]$.

Hence,

$$\tilde{H}_\alpha = \begin{cases} \left[\frac{\alpha}{r}(\beta_2-\beta_1)+\beta_1, -\frac{\alpha}{s}(\beta_6-\beta_5)+\beta_6\right] & \text{for } \alpha \in [0, 0.5] \\ \left[\frac{\alpha-r}{1-r}(\beta_3-\beta_2)+\beta_2, -\frac{\alpha-s}{1-s}(\beta_5-\beta_4)+\beta_5\right] & \text{for } \alpha \in [0.5, 1]. \end{cases} \tag{7}$$

If we consider $r = 0.5$ and $s = 0.5$, then α-cut of $\tilde{H}$ becomes

$$\tilde{H}_\alpha = \begin{cases} [2\alpha(\beta_2-\beta_1)+\beta_1, -2\alpha(\beta_6-\beta_5)+\beta_6] & \text{for } \alpha \in [0, 0.5] \\ [(2\alpha-1)(\beta_3-\beta_2)+\beta_2, -(2\alpha-1)(\beta_5-\beta_4)+\beta_5] & \text{for } \alpha \in [0.5, 1]. \end{cases} \tag{8}$$

Example 1. Let us consider $\tilde{H} = \{(2,3,4,7,8,9); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(4,6,8,14,16,18); \mu_{\tilde{G}}\}$ are two hexagonal fuzzy numbers and their membership functions are continuous. Then α-cut of two fuzzy numbers are $\tilde{H}$ and $\tilde{G}$ are respectively $\tilde{H}_\alpha = [2\alpha+2, -2\alpha+9]$ and $\tilde{G}_\alpha = [4\alpha+4, -4\alpha+18]$ where $\alpha \in [0,1]$.

2.5. α-cut operations

Let us consider $\tilde{H} = \{(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5,\beta_6); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5,\zeta_6); \mu_{\tilde{G}}\}$ are two Hexagonal fuzzy numbers and membership functions of $\tilde{H}$ and $\tilde{G}$ are continuous. Also let $\tilde{H}_\alpha$ and $\tilde{G}_\alpha$ are two α-cut of HFN $\tilde{H}$ and $\tilde{G}$ respectively. α-cut operations are described on [45–47].

Then *addition* of two α-cuts $\tilde{H}_\alpha$ and $\tilde{G}_\alpha$ is $\tilde{H}_\alpha + \tilde{G}_\alpha =$

$$\begin{cases} [2\alpha(\beta_2-\beta_1)+\beta_1, -2\alpha(\beta_6-\beta_5)+\beta_6] \\ \quad +[2\alpha(\zeta_2-\zeta_1)+\zeta_1, -2\alpha(\zeta_6-\zeta_5)+\zeta_6] & \text{for } \alpha \in [0, 0.5] \\ [(2\alpha-1)(\beta_3-\beta_2)+\beta_2, -(2\alpha-1)(\beta_5-\beta_4)+\beta_5] \\ \quad +[(2\alpha-1)(\zeta_3-\zeta_2)+\zeta_2, -(2\alpha-1)(\zeta_5-\zeta_4)+\zeta_5] & \text{for } \alpha \in [0.5, 1] \end{cases} \tag{9}$$

We again consider $\tilde{H} = \{(2,3,4,7,8,9); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(4,6,8,14,16,18); \mu_{\tilde{G}}\}$ are two Hexagonal fuzzy numbers with membership functions are continuous. Then addition of α-cut of two fuzzy numbers is $\tilde{H}_\alpha + \tilde{G}_\alpha = [2\alpha + 2, -2\alpha + 9] + [4\alpha + 4, -4\alpha + 18] = [6\alpha + 6, -6\alpha + 27]$ where $\alpha \in [0,1]$.

If $\alpha = 0$ then $\tilde{H}_0 + \tilde{G}_0 = [6,27]$, and if $\alpha = 0.5$ then $\tilde{H}_{0.5} + \tilde{G}_{0.5} = [9,24]$ and if $\alpha = 1$ then $\tilde{H}_1 + \tilde{G}_1 = [12,23]$. Hence $\tilde{H}_\alpha + \tilde{G}_\alpha = \{(6,9,12,23,24,27); \mu_{\tilde{H}} + \mu_{\tilde{G}}\}$, this imply that all points coincide with the sum of two hexagonal fuzzy numbers.

Then *subtraction* of two α-cuts $\tilde{H}_\alpha$ and $\tilde{G}_\alpha$ is $\tilde{H}_\alpha - \tilde{G}_\alpha =$

$$\begin{cases} [2\alpha(\beta_2 - \beta_1) + \beta_1, -2\alpha(\beta_6 - \beta_5) + \beta_6] & \\ \quad -[2\alpha(\zeta_2 - \zeta_1) + \zeta_1, -2\alpha(\zeta_6 - \zeta_5) + \zeta_6] & \text{if } \alpha \in [0, 0.5] \\ [(2\alpha - 1)(\beta_3 - \beta_2) + \beta_2, -(2\alpha - 1)(\beta_5 - \beta_4) + \beta_5] & \\ \quad -[(2\alpha - 1)(\zeta_3 - \zeta_2) + \zeta_2, -(2\alpha - 1)(\zeta_5 - \zeta_4) + \zeta_5] & \text{if } \alpha \in [0.5, 1]. \end{cases} \tag{10}$$

For same example $\tilde{H} = \{(2,3,4,7,8,9); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(4,6,8,14,16,18); \mu_{\tilde{G}}\}$ are two Hexagonal fuzzy numbers with membership functions are continuous. Then subtraction of α-cut of two fuzzy numbers is $\tilde{H}_\alpha - \tilde{G}_\alpha = [2\alpha + 2, -2\alpha + 9] - [4\alpha + 4, -4\alpha + 18] = [-2\alpha - 2, 2\alpha - 9]$ where $\alpha \in [0,1]$.

If $\alpha = 0$ then $\tilde{H}_0 - \tilde{G}_0 = [-2, -9]$, and if $\alpha = 0.5$ then $\tilde{H}_{0.5} - \tilde{G}_{0.5} = [-3 - 8]$ and if $\alpha = 1$ then $\tilde{H}_1 - \tilde{G}_1 = [-4, -7]$. Hence $\tilde{H}_\alpha - \tilde{G}_\alpha = \{(-2,-3,-4,-7,-8,-9); \mu_{\tilde{H}} - \mu_{\tilde{G}}\}$, this imply that all value are same with the subtraction of two HFNs.

Then *multiplication* of two α-cuts $\tilde{H}_\alpha$ and $\tilde{G}_\alpha$ is $\tilde{H}_\alpha \times \tilde{G}_\alpha =$

$$\begin{cases} [2\alpha(\beta_2 - \beta_1) + \beta_1, -2\alpha(\beta_6 - \beta_5) + \beta_6] & \\ \quad \times[2\alpha(\zeta_2 - \zeta_1) + \zeta_1, -2\alpha(\zeta_6 - \zeta_5) + \zeta_6] & \text{if } \alpha \in [0, 0.5] \\ [(2\alpha - 1)(\beta_3 - \beta_2) + \beta_2, -(2\alpha - 1)(\beta_5 - \beta_4) + \beta_5] & \\ \quad \times[(2\alpha - 1)(\zeta_3 - \zeta_2) + \zeta_2, -(2\alpha - 1)(\zeta_5 - \zeta_4) + \zeta_5] & \text{if } \alpha \in [0.5, 1]. \end{cases} \tag{11}$$

We again consider $\tilde{H} = \{(2,3,4,7,8,9); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(4,6,8,14,16,18); \mu_{\tilde{G}}\}$ are two Hexagonal fuzzy numbers with membership functions are continuous. Then multiplication of α-cut of two fuzzy numbers is $\tilde{H}_\alpha \times \tilde{G}_\alpha = [2\alpha + 2, -2\alpha + 9] \times [4\alpha + 4, -4\alpha + 18] = [(2\alpha + 2) \times (-4\alpha + 4), (-2\alpha + 9) \times (-4\alpha + 18)]$ where $\alpha \in [0,1]$.

If $\alpha = 0$ then $\tilde{H}_0 \times \tilde{G}_0 = [6,162]$, and if $\alpha = 0.5$ then $\tilde{H}_{0.5} \times \tilde{G}_{0.5} = [18,128]$ and if $\alpha = 1$ then $\tilde{H}_1 \times \tilde{G}_1 = [32,98]$. Hence $\tilde{H}_\alpha \times \tilde{G}_\alpha = \{(6,18,32,98,128,162); \mu_{\tilde{H}} \times \mu_{\tilde{g}}\}$, this imply that all points coincide with the multiplication of two hexagonal fuzzy numbers.

2.6. Defuzzification methods of HFN

Defuzzification is the procedure to produce a quantifiable result in crisp logic from fuzzy set and it's membership function. It is generally needed in fuzzy control systems. Defuzzification and fuzzification are opposite process to convert fuzzy set to crisp set and vice versa respectively. There exist several defuzzification methods, but the common and useful methods described as follows:

2.6.1. Centroid-based method (CBM) of a hexagonal fuzzy number:

Let $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6); \mu_{\tilde{H}}\}$ is a hexagonal fuzzy number with $\mu_{\tilde{H}} = 0.5$. This method described and applied in the paper Ghosh *et al.* [11]. HFN is divided into four sub region; two triangle and two trapezium. Furthermore, one trapezium can be divided into three sub parts; two triangle and one rectangle. At the end, summation of all triangles and rectangles are excuted to get the centroid formulae of HFN. The method is illustrated below

- The centroid of triangle $\triangle$ AIB is $\left(\frac{\beta_1+\beta_2+\beta_3}{3}, \frac{r}{3}\right)$
- The centroid of triangle $\triangle$ FEJ is $\left(\frac{\beta_4+\beta_5+\beta_6}{3}, \frac{r}{3}\right)$

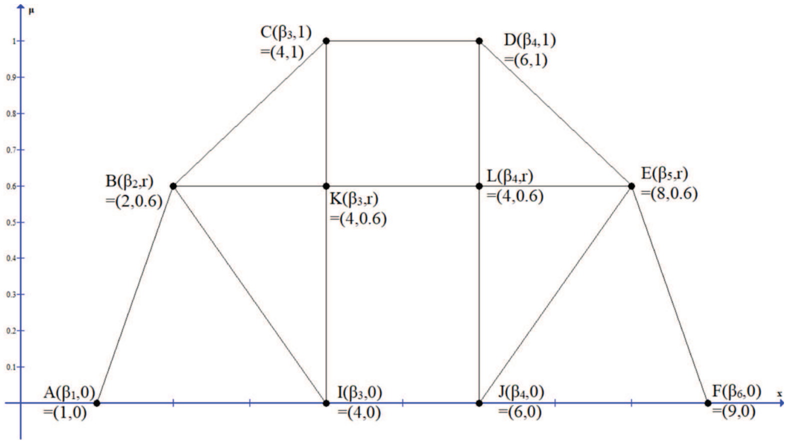

FIGURE 4. $\tilde{H}$ is a hexagonal fuzzy number (HFN), divided into six triangles and two rectangles is visualise in this picture.

- The centroid of trapezium $\square$ IJEB is
  - The centroid of triangle $\triangle$ IKB is $\left(\frac{\beta_2+2\beta_3}{3}, \frac{2r}{3}\right)$
  - The centroid of triangle $\triangle$ JEL is $\left(\frac{2\beta_4+\beta_5}{3}, \frac{2r}{3}\right)$
  - The centroid of rectangle $\square$ IJLK is $\left(\frac{\beta_3+\beta_4}{2}, \frac{r}{2}\right)$

  Therefore, the trapezium $\square$ IJEB is $\left(\frac{2\beta_2+7\beta_3+7\beta_4+2\beta_5}{6}, \frac{11r}{6}\right)$
- The centroid of trapezium $\square$ BEDC is $\left(\frac{2\beta_2+7\beta_3+7\beta_4+2\beta_5}{6}, \frac{11r+7}{6}\right)$ by similar way.

Therefore, the centroid of this HFN is

$$\mathrm{CBM}(\tilde{H}) = \left(\frac{3\beta_1+3\beta_2+10\beta_3+10\beta_4+5\beta_5+3\beta_6}{34}, \frac{26r+7}{6}\right). \tag{12}$$

This is the defuzzified value of the hexagonal fuzzy number $\tilde{H}$ and geometrically represents at Figure 4.

2.7. Arithmetic operation on HFN

Arithmetic operation on hexagonal fuzzy numbers (HFN) plays significant role in the theory of hexagonal fuzzy numbers and it's application [48]. Therefore it is an important branch of research. Different operations exist in the field of hexagonal fuzzy number (HFN) and it is very useful and popular. In this section, we define some important operations such as addition, subtraction, multiplication and division on HFN.

Definition 9 (Addition of HFN)**.** If $\tilde{H} = \{(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5,\beta_6;r,s);\mu_{\tilde{H}}\}$ and $\tilde{G} = \{(\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5,\zeta_6;t,u);\mu_{\tilde{G}}\}$ are two hexagonal fuzzy number (HFN), then the addition of $\tilde{H}$ and $\tilde{G}$ is define as $\tilde{H} \oplus \tilde{G} = \{(\beta_1+\zeta_1,\beta_2+\zeta_2,\beta_3+\zeta_3,\beta_4+\zeta_4,\beta_5+\zeta_5,\beta_6+\zeta_6;v,w);\mu_{\tilde{H}\oplus\tilde{G}}\}$ whose membership function is given by $\mu_{\tilde{H}\oplus\tilde{G}}(\phi) =$

$$\begin{cases} 0 & \text{if } \phi \in (-\infty, \beta_1+\zeta_1] \\ v\left(\frac{\phi-(\beta_1+\zeta_1)}{(\beta_2+\zeta_2)-(\beta_1+\zeta_1)}\right) & \text{if } \phi \in [(\beta_1+\zeta_1),(\beta_2+\zeta_2)] \\ v+(1-v)\left(\frac{\phi-(\beta_2+\zeta_2)}{(\beta_3+\zeta_3)-(\beta_2+\zeta_2)}\right) & \text{if } \phi \in [(\beta_2+\zeta_2),(\beta_3+\zeta_3)] \\ 1 & \text{if } \phi \in [(\beta_3+\zeta_3),(\beta_4+\zeta_4)] \\ w+(1-w)\left(\frac{(\beta_5+\zeta_5)-x}{(\beta_5+\zeta_5)-(\beta_4+\zeta_4)}\right) & \text{if } \phi \in [(\beta_4+\zeta_4),(\beta_5+\zeta_5)] \\ w\left(\frac{(\beta_6+\zeta_6)-\phi}{(\beta_6+\zeta_6)-(\beta_5+\zeta_5)}\right) & \text{if } \phi \in [(\beta_5+\zeta_5),(\beta_6+\zeta_6)] \\ 0 & \text{if } \phi \in [(\beta_6+\zeta_6),\infty) \end{cases} \tag{13}$$

where $v = r + t - rt$, $w = s + u - su$ and $\mu_{\tilde{H}\oplus\tilde{G}} = \mu_{\tilde{H}} + \mu_{\tilde{G}} - \mu_{\tilde{H}}\mu_{\tilde{G}}$.

Definition 10 (Subtraction of HFN). If $\tilde{H} = \{(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5,\beta_6;r,s);\mu_{\tilde{H}}\}$ and $\tilde{G} = \{(\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5,\zeta_6;t,u);\mu_{\tilde{G}}\}$ are two hexagonal fuzzy number (HFN), then the subtraction of $\tilde{H}$ and $\tilde{G}$ is define as $\tilde{H}\ominus\tilde{G} = \left\{(\beta_1-\zeta_6,\beta_2-\zeta_5,\beta_3-\zeta_4,\beta_4-\zeta_3,\beta_5-\zeta_2,\beta_6-\zeta_1;v,w);\mu_{\tilde{H}\ominus\tilde{G}}\right\}$ whose membership function is given by $\mu_{\tilde{H}\ominus\tilde{G}}(\phi) =$

$$\begin{cases} 0 & \text{if } \phi \in (-\infty, \beta_1-\zeta_6] \\ v\left(\frac{\phi-(\beta_1-\zeta_6)}{(\beta_2-\zeta_5)-(\beta_1-\zeta_6)}\right) & \text{if } \phi \in [(\beta_1-\zeta_6),(\beta_2-\zeta_5)] \\ v+(1-v)\left(\frac{\phi-(\beta_2-\zeta_5)}{(\beta_3-\zeta_4)-(\beta_2-\zeta_5)}\right) & \text{if } \phi \in [(\beta_2-\zeta_5),(\beta_3-\zeta_4)] \\ 1 & \text{if } \phi \in [(\beta_3-\zeta_4),(\beta_4-\zeta_3)] \\ w+(1-w)\left(\frac{(\beta_5-\zeta_2)-\phi}{(\beta_5-\zeta_2)-(\beta_4-\zeta_3)}\right) & \text{if } \phi \in [(\beta_4-\zeta_3),(\beta_5-\zeta_2)] \\ w\left(\frac{(\beta_6-\zeta_1)-\phi}{(\beta_6-\zeta_1)-(\beta_5-\zeta_2)}\right) & \text{if } \phi \in [(\beta_5-\zeta_2),(\beta_6-\zeta_1)] \\ 0 & \text{if } \phi \in [(\beta_6-\zeta_1),\infty) \end{cases} \tag{14}$$

where $v = r + u - ru$, $w = s + t - st$ and $\mu_{\tilde{H}\ominus\tilde{G}} = \mu_{\tilde{H}} + \mu_{\tilde{G}} - \mu_{\tilde{H}}\mu_{\tilde{G}}$.

Definition 11 (Scalar multiplication of HFN). If $\tilde{H} = \{(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5,\beta_6;r,s);\mu_{\tilde{H}}\}$ is a hexagonal fuzzy number (HFN) and k is taken as positive real constant, then the scalar multiplication of $\tilde{H}$ by k is define by

$$k\tilde{H} = \{(k\beta_1,k\beta_2,k\beta_3,k\beta_4,k\beta_5,k\beta_6;r,s);\mu_{\tilde{H}}\}. \tag{15}$$

Scalar multiplication also true for negative real constant in similar way.

Definition 12 (Multiplication of HFN). If $\tilde{H} = \{(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5,\beta_6;r,s);\mu_{\tilde{H}}\}$ and $\tilde{G} = \{(\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5,\zeta_6;t,u);\mu_{\tilde{G}}\}$ are two hexagonal fuzzy number (HFN), then the multiplication of $\tilde{H}$ and

$\tilde{G}$ is define as $\tilde{H} \otimes \tilde{G}(\phi) =$

$$\begin{cases} \{(\min(\beta_1\zeta_6, \beta_6\zeta_6), \min(\beta_2\zeta_5, \beta_5\zeta_5), \min(\beta_3\zeta_4, \beta_4\zeta_4), \max(\beta_4\zeta_3, \beta_3\zeta_3), \\ \max(\beta_5\zeta_2, \beta_2\zeta_2), \max(\beta_6\zeta_1, \beta_1\zeta_1), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_6 \leq 0 \\ \{(\min(\beta_1\zeta_6, \beta_6\zeta_1), \min(\beta_2\zeta_5, \beta_5\zeta_5), \min(\beta_3\zeta_4, \beta_4\zeta_4), \max(\beta_4\zeta_3, \beta_3\zeta_3), \\ \max(\beta_5\zeta_2, \beta_2\zeta_2), \max(\beta_6\zeta_6, \beta_1\zeta_1), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_5 \leq 0, \beta_6 \geq 0 \\ \{(\min(\beta_1\zeta_6, \beta_6\zeta_1), \min(\beta_2\zeta_5, \beta_5\zeta_2), \min(\beta_3\zeta_4, \beta_4\zeta_4), \max(\beta_4\zeta_3, \beta_3\zeta_3), \\ \max(\beta_5\zeta_5, \beta_2\zeta_2), \max(\beta_6\zeta_6, \beta_1\zeta_1), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_4 \leq 0, \beta_5 \geq 0 \\ \{(\min(\beta_1\zeta_6, \beta_6\zeta_1), \min(\beta_2\zeta_5, \beta_5\zeta_2), \min(\beta_3\zeta_4, \beta_4\zeta_3), \max(\beta_4\zeta_4, \beta_3\zeta_3), \\ \max(\beta_5\zeta_5, \beta_2\zeta_2), \max(\beta_6\zeta_6, \beta_1\zeta_1), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_3 \leq 0, \beta_4 \geq 0 \\ \{(\min(\beta_1\zeta_6, \beta_6\zeta_1), \min(\beta_2\zeta_5, \beta_5\zeta_2), \min(\beta_3\zeta_3, \beta_4\zeta_3), \max(\beta_4\zeta_4, \beta_3\zeta_4), \\ \max(\beta_5\zeta_5, \beta_2\zeta_2), \max(\beta_6\zeta_6, \beta_1\zeta_1), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_2 \leq 0, \beta_3 \geq 0 \\ \{(\min(\beta_1\zeta_6, \beta_6\zeta_1), \min(\beta_2\zeta_2, \beta_5\zeta_2), \min(\beta_3\zeta_3, \beta_4\zeta_3), \max(\beta_4\zeta_4, \beta_3\zeta_4), \\ \max(\beta_5\zeta_5, \beta_2\zeta_5), \max(\beta_6\zeta_6, \beta_1\zeta_1), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_1 \leq 0, \beta_2 \geq 0 \\ \{(\min(\beta_1\zeta_1, \beta_6\zeta_1), \min(\beta_2\zeta_2, \beta_5\zeta_2), \min(\beta_3\zeta_3, \beta_4\zeta_3), \max(\beta_4\zeta_4, \beta_3\zeta_4), \\ \max(\beta_5\zeta_5, \beta_2\zeta_5), \max(\beta_6\zeta_6, \beta_1\zeta_6), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_1 \geq 0. \end{cases} \tag{16}$$

Definition 13 (Division of HFN)**.** If $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6; t, u); \mu_{\tilde{G}}\}$ are two hexagonal fuzzy number (HFN) with $\tilde{H} > 0$ & $\tilde{G} > 0$, then the division of $\tilde{H}$ and $\tilde{G}$ is define as $\tilde{H} \oslash \tilde{G} = \left\{(\beta_1/\zeta_6, \beta_2/\zeta_5, \beta_3/\zeta_4, \beta_4/\zeta_3, \beta_5/\zeta_2, \beta_6/\zeta_1; v, w); \mu_{\tilde{H}\oslash\tilde{G}}\right\}$ whose membership function is given by

$$\mu_{\tilde{H}\oslash\tilde{G}}(\phi) = \begin{cases} 0 & \text{if } \phi \in (-\infty, \beta_1/\zeta_6] \\ v\left(\frac{\phi-(\beta_1/\zeta_6)}{(\beta_2/\zeta_5)-(\beta_1/\zeta_6)}\right) & \text{if } \phi \in [(\beta_1/\zeta_6), (\beta_2/\zeta_5)] \\ v + (1-v)\left(\frac{\phi-(\beta_2/\zeta_5)}{(\beta_3/\zeta_4)-(\beta_2/\zeta_5)}\right) & \text{if } \phi \in [(\beta_2/\zeta_5), (\beta_3/\zeta_4)] \\ 1 & \text{if } \phi \in [(\beta_3/\zeta_4), (\beta_4/\zeta_3)] \\ w + (1-w)\left(\frac{(\beta_5/\zeta_2)-\phi}{(\beta_5/\zeta_2)-(\beta_4/\zeta_3)}\right) & \text{if } \phi \in [(\beta_4/\zeta_3), (\beta_5/\zeta_2)] \\ w\left(\frac{(\beta_6/\zeta_1)-\phi}{(\beta_6/\zeta_1)-(\beta_5/\zeta_2)}\right) & \text{if } \phi \in [(\beta_5/\zeta_2), (\beta_6/\zeta_1)] \\ 0 & \text{if } \phi \in [(\beta_6/\zeta_1), \infty) \end{cases} \tag{17}$$

where $v = ru$, $w = st$ and $\mu_{\tilde{H}\oslash\tilde{G}} = \mu_{\tilde{H}}\mu_{\tilde{G}}$.

If $\tilde{H} > 0$ & $\tilde{G} < 0$, then the division of $\tilde{H}$ and $\tilde{G}$ is define as $\tilde{H} \oslash \tilde{G} = \left\{(\beta_6/\zeta_1, \beta_5/\zeta_2, \beta_4/\zeta_3, \beta_3/\zeta_4, \beta_2/\zeta_5, \beta_1/\zeta_6; v, w); \mu_{\tilde{H}\oslash\tilde{G}}\right\}$ whose membership function is given by

$$\mu_{\tilde{H}\oslash\tilde{G}}(\phi) = \begin{cases} 0 & \text{if } \phi \in (-\infty, \beta_6/\zeta_1] \\ v\left(\frac{\phi-(\beta_6/\zeta_1)}{(\beta_5/b_2)-(\beta_6/\zeta_1)}\right) & \text{if } \phi \in [(\beta_6/\zeta_1), (\beta_5/\zeta_2)] \\ v + (1-v)\left(\frac{\phi-(\beta_5/\zeta_2)}{(\beta_4/\zeta_3)-(\beta_5/\zeta_2)}\right) & \text{if } \phi \in [(\beta_5/\zeta_2), (\beta_4/\zeta_3)] \\ 1 & \text{if } \phi \in [(\beta_4/\zeta_3), (\beta_3/\zeta_4)] \\ w + (1-w)\left(\frac{(\beta_2/\zeta_5)-\phi}{(\beta_2/\zeta_5)-(\beta_3/\zeta_4)}\right) & \text{if } \phi \in [(\beta_3/\zeta_4), (\beta_2/\zeta_5)] \\ w\left(\frac{(\beta_1/\zeta_6)-\phi}{(\beta_1/\zeta_6)-(\beta_2/\zeta_5)}\right) & \text{if } \phi \in [(\beta_2/\zeta_5), (\beta_1/\zeta_6)] \\ 0 & \text{if } \phi \in [(\beta_1/\zeta_6), \infty) \end{cases} \tag{18}$$

where $v = st$, $w = ru$ and $\mu_{\tilde{H}\oslash\tilde{G}} = \mu_{\tilde{H}}\mu_{\tilde{G}}$.

Definition 14 (Inverse of HFN)**.** If $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}}\}$ is a hexagonal fuzzy number (HFN) with $0 \notin \tilde{H}$ (*i.e.*, $0 \notin [\beta_1, \beta_6]$), then inverse of $\tilde{H}$ define as

$$\tilde{H}^{-1} = \{(1/\beta_6, 1/\beta_5, 1/\beta_4, 1/\beta_3, 1/\beta_2, 1/\beta_1; v, w); \mu_{\tilde{H}^{-1}}\} \tag{19}$$

where $v = s$, $w = r$ and $\mu_{\tilde{H}^{-1}} = \mu_{\tilde{H}}$.

Also, if $0 \in \tilde{H}$ then inverse is not define.

Definition 15 (Identical Equality of two HFN)**.** Let $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6; t, u); \mu_{\tilde{G}}\}$ are two hexagonal fuzzy number (HFN), then they are identically equal if $\beta_1 = \zeta_1$, $\beta_2 = \zeta_2$, $\beta_3 = \zeta_3$, $\beta_4 = \zeta_4$, $\beta_5 = \zeta_5$, $\beta_6 = \zeta_6$, $r = t$, $s = u$ and $\mu_{\tilde{H}} = \mu_{\tilde{G}}$.

Numerical example: Numerical example has been discussed below to illustrate the arithmetic operations of Hexagonal fuzzy numbers.

Example 2. Let $\tilde{H} = \{(3, 4, 6, 7.5, 9, 10; 0.25, 0.7); 1\}$ and $\tilde{G} = \{(2, 2.5, 3.5, 5, 6, 7; 0.4, 0.5); 1\}$ are two hexagonal fuzzy numbers and $k = 2(> 0)$ be a scalar. Then arithmetic operation on HFN gives

(1) Addition: $\tilde{H} \oplus \tilde{G} = \{(5, 6.5, , 9.5, 12.5, 15, 17; 0.55, 0.85); 1\}$
(2) Subtraction: $\tilde{H} \ominus \tilde{G} = \{(-4, -2, 1, 4, 6.5, 8; 0.625, 0.82); 1\}$
(3) Scalar multiplication: $2\tilde{H} = 2 \times \tilde{H} = \{(6, 8, 12, 15, 18, 20; 0.25, 0.7); 1\}$
(4) Multiplication: $\tilde{H} \otimes \tilde{G} = \{(6, 10, 21, 37.5, 54, 70; 0.1, 0.35); 1\}$
(5) Division: $\tilde{H} \oslash \tilde{G} = \{(3/7, 2/3, 6/5, 15/7, 18/5, 5; 0.125, 0.28); 1\}$
(6) Inverse: $\tilde{H}^{-1} = \{(1/10, 1/9, 2/15, 1/6, 1/4, 1/3; 0.7, 0.25); 1\}$

Example 3. Let $\tilde{A} = \{(1, 4, 6, 7, 8, 10; 0.6, 0.4); 0.5\}$ and $\tilde{B} = \{(-15, -13, -10, -8, -4, -2; 0.2, 0.8); 0.5\}$ are two hexagonal fuzzy numbers and $k = -2(< 0)$ be a scalar. Then arithmetic operation on HFN gives

(1) Addition: $\tilde{A} \oplus \tilde{B} = \{(-14, -9, -4, -1, 4, 8; 0.68, 0.88); 0.75\}$
(2) Subtraction: $\tilde{A} \ominus \tilde{B} = \{(3, 8, 14, 17, 21, 25; 0.92, 0.52); 0.75\}$
(3) Scalar multiplication: $-2\tilde{B} = (-2) \times \tilde{B} = \{(4, 8, 16, 20, 26, 30; 0.2, 0.8); 0.5\}$
(4) Multiplication: $\tilde{A} \otimes \tilde{B} = \{(-150, -104, -70, -48, -16, -2; 0.12, 0.32); 0.25\}$
(5) Division: $\tilde{A} \oslash \tilde{B} = \{(-2/3, -8/13, -7/10, -3/4, -1, -1/2; 0.08, 0.48); 0.25\}$
(6) Inverse: $\tilde{B}^{-1} = \{(-1/2, -1/4, -1/8, -1/10, -1/13, -1/15; 0.8, 0.2); 0.5\}$

3. Hexagonal fuzzy multi sub criterion based decision making

3.1. Fuzzy Analytic Hierarchy Process (FAHP) Method

Analytic hierarchy process (AHP) is a famous mathematical tool of optimization of alternatives which is used in Multi Criteria Decision Making (MCDM). This method was first developed by Satty [49] and Wind and Saaty [50]. It is used explicitly for obtaining the factors and sub-factors weight. AHP gives a scientific solution to real life problems. This method help decision makers (DMs) to resolve complex problems with heuristic solution. The comparison of factors and sub-factors, by giving preference in crisp value can be considered as a complex assignment for DMs, thus FAHP methodology captures the blurriness of the problem. The determination of factors and sub-factors weights are very important for the customers in a restaurant. AHP works with a problem hierarchy, where a comparison matrix is constructed to represent subjective judgments regarding factors and sub-factors. In this paper, FAHP is taken instead of AHP, keeping in mind the fuzzy logic which allows the DMs in the evaluation of the well optimized result. The step of FAHP are given described below:

(i) Construction of comparison matrix in term of hexagonal fuzzy number (HFN) by a group of decision experts.
Let a group of 'N' decision-makers assigned for the comparison of factors and sub-factors. Let each DM express their preference in the pairwise comparison of factors and sub-factors. Thus, 'n' set of matrices are obtained, $T_n = \{t_{pqn}\}$.
Where $t_{pqn} = \left(\tilde{a}_{pqn}, \tilde{b}_{pqn}, \tilde{c}_{pqn}, \tilde{d}_{pqn}, \tilde{e}_{pqn}, \tilde{f}_{pqn}\right)$ denotes the HFN of p factor to q factor as expressed by the 'n' DM and $p = 1, 2, \ldots, i;\ q = 1, 2, \ldots, j$.

$$\begin{cases} a_{pq} = \min_{n=1,2,\ldots,N} \tilde{a}_{pqn} \\ b_{pq} = \min_{n=1,2,\ldots,N} \tilde{b}_{pqn} \\ c_{pq} = \sqrt[N]{\prod_{n=1}^{N} \tilde{c}_{pqn}} \\ d_{pq} = \sqrt[N]{\prod_{n=1}^{N} \tilde{d}_{pqn}} \\ e_{pq} = \max_{n=1,2,\ldots,N} \tilde{e}_{pqn} \\ f_{pq} = \max_{n=1,2,\ldots,N} \tilde{f}_{pqn} \end{cases} \tag{20}$$

(ii) Defuzzification of HFN:
Defuzzification of the hexagonal fuzzy number (HNF) by the centroid-based method (CBM) used this paper. Thus using equation (12), convert a fuzzy number to a crisp value.

(iii) Normalization of the defuzzied matrix:

$$\begin{cases} S_q = \sum_{p=1}^{i} V_{pq} \\ U_{pq} = \frac{V_{pq}}{S_q} \end{cases} \tag{21}$$

where $p = 1, 2, \ldots, i;\ q = 1, 2, \ldots, j$. This normalization makes the sum of the weights equal to one.

(iv) Estimation of factors and sub-factors weights:

$$E = \frac{N\text{th root value}}{\sum N\text{th root}}. \tag{22}$$

(v) To test the Consistence Index (C.I.) of the matrix:

$$C.I. = \frac{\alpha_{\max} - j}{j - 1} \tag{23}$$

where j denotes the size of the matrix.

(vi) Determination of Consistency Ratio (C.R.):

$$\text{C.R.} = \frac{\text{C.I.}}{\text{R.I.}} \tag{24}$$

where R.I. denote Random Index and its value depends on the size of the matrix n.

The assessment of C.R. ≤ 0.1 is acceptable and indicates that the weights obtained are consistent.

3.2. Determination of hexagonal fuzzy weights of factors

To obtain the HFN weight of factors and sub- factors, we refer Ghosh *et al.* [11] and the process of determining the HFN weight is discussed below:

(i) Determine the geometric mean of the HFN by using

$$a_{rt} = \left(\prod_{s=1}^{k} y_{rst}\right)^{\frac{1}{k}} \tag{25}$$

where $y_{rst} \in \tilde{H}$, $r = 1, 2, \ldots, k$; $s = 1, 2, \ldots, k$ and $t = 1, 2, \ldots, 6$.

(ii) Summing the each column of geometric mean criterion matrix.

$$S_t = \sum_{r=1}^{k} a_{rt} \tag{26}$$

where bound of r and t are same as previous.

(iii) Then find the inverse of S_t using the equation (19) and then arranging in increasing order, S'_t(let).

(iv) The hexagonal fuzzy weights of factors by the given equation:

$$W_t = a_{rt} \times S'_t. \tag{27}$$

(v) Calculation of global HFN sub-factors weight are determined by the product of factors HFN weight with the respective sub-factor HFN weight.

3.3. Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) approach

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) Multi Criteria Decision Making (MCDM) tool is a widely used technique developed by Hwang and Yoon [51]. This method also use in different field like Best Employees selection [52], Medicine selection [12], Creating priority group for Vaccine [13] etc. The TOPSIS method is taken as a distance measure technique in which the optimal alternative is nearest to the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS). The linguistic terms assigned by human choice of decisions can be reflected by HFN. This approach is convenient for handling the uncertainty of the situation involving multiple factors and sub-factors. Thus, for the factor attracting the customers in a restaurant, the classical MCDM method TOPSIS is integrated with Fuzzy number *i.e.* FTOPSIS to capture the problem in an efficient way. The steps of FTOPSIS are described below:

(i) Construction of the decision matrix by the help of decision experts in terms of linguistic terms. The linguistic terms are then converted to a HFN.
Let i number of restaurants and j number of factors. Let N denotes the number of decision makers (DMs). Then evaluate the value of $\tilde{a}_{pq}$, $\tilde{b}_{pq}$, $\tilde{c}_{pq}$, $\tilde{d}_{pq}$, $\tilde{e}_{pq}$ and $\tilde{f}_{pq}$ by equation (20) and the value of r_{pq}, s_{pq} find by:

$$\begin{cases} r_{pq} = \min_{n=1,2,3,\ldots,N} r_{pqn} \\ s_{pq} = \min_{n=1,2,3,\ldots,N} s_{pqn} \end{cases} \tag{28}$$

(ii) To evaluate the normalized HFN fuzzy decision matrix:

$$\begin{aligned} \tilde{N} &= [n_{pq}]_{ij}, \qquad p = 1, 2, 3, \ldots, i;\ q = 1, 2, 3, \ldots, j \\ N_{pq}^{B} &= \left\langle \left(\frac{a_{pq}}{f^*}, \frac{b_{pq}}{f^*}, \frac{c_{pq}}{f^*}, \frac{d_{pq}}{f^*}, \frac{e_{pq}}{f^*}, \frac{f_{pq}}{f^*} \right); r_{pq}, s_{pq} \right\rangle \quad d \in \text{B.A},\ f^* = \max f_{pq} \\ N_{pq}^{NB} &= \left\langle \left(\frac{a^*}{f_{pq}}, \frac{a^*}{e_{pq}}, \frac{a^*}{d_{pq}}, \frac{a^*}{c_{pq}}, \frac{a^*}{b_{pq}}, \frac{a^*}{a_{pq}} \right); r_{pq}, s_{pq} \right\rangle \quad d \in \text{N.B.A},\ a^* = \min a_{pq} \end{aligned} \tag{29}$$

where B.A and N.B.A signifies the benefit attributes and non-benefit attributes, respectively.

(iii) To evaluate the weighted fuzzy normalized matrix, the sub-factors' fuzzy weights describe on equation (27) are multiplied with the normalized fuzzy value:

$$WN = [P_{pq}]_{ij} \tag{30}$$

where $P_{pq} = N_{pq} * \tilde{W}_q$, $p = 1, 2, 3, \ldots, i$; $q = 1, 2, 3, \ldots, j$.

(iv) Calculate the fuzzy positive ideal solution (FPIS) (PIS^+) and fuzzy negative ideal solution (FNIS) (NIS^-), where a_p^+ denotes the maximum value of a_{pq} and a_p^- denotes the minimum value of a_{pq}:

$$\left.\begin{array}{l}\mathrm{PIS}^+ = \{a_1^+, a_2^+, \ldots, a_j^+\} = \{(\max\, a_{pq}|q \in M_B), (\min\, a_{pq}|q \in M_{NB})\} \\ \mathrm{NIS}^- = \{a_1^-, a_2^-, \ldots, a_j^-\} = \{(\min\, a_{pq}|q \in M_B), (\max\, a_{pq}|q \in M_{NB})\}\end{array}\right\} \tag{31}$$

where M_B denotes the benefit attributes and M_{NB} denotes the non-benefit attributes.

(v) Calculation of the distance measure of all alternatives from the PIS and NIS. The two Euclidean distances for individual alternatives can be calculated as follows:

$$\left.\begin{array}{l}L_p^+ = \sum_{q=1}^{j} d(P_{pq}, q_p^+),\ p = 1,2,3,\ldots,i \\ L_p^- = \sum_{q=1}^{j} d(P_{pq}, q_p^-),\ p = 1,2,3,\ldots,i\end{array}\right\} \tag{32}$$

where $d(.,.)$ denotes the Euclidean distance between two fuzzy numbers.

(vi) Determination of the relative closeness to the ideal alternatives:

$$R_p = \frac{L_p^-}{L_p^+ + L_p^-} \tag{33}$$

where $p = 1,2,3,\ldots,i$.

(vii) Rank the alternatives:
The alternatives are ranked based on the score obtained by R_p. The larger value of R_p signifies the better alternatives.

3.4. Fuzzy Complex Proportional Assessment (FCOPRAS) approach

The COmplex PRoportional ASsessment (COPRAS) was first introduced by Zavadskas, Kalklauskas and Sarka [53]. An extended representation of COPRAS is Fuzzy COPRAS (FCOPRAS) which is used for ranking of the alternatives in various decision making problem [54]. This method is based on stepwise ranking and evaluation of the alternative in reference to utility degree and significance. Earlier COPRAS method were applied by Ghosh *et al.* in site selection [11], Fouladgar *et al.* in property management [55], economy by Narayanamoothy *et al.* [56], Evaluating the potential capability of air cargo sector Tolga and Durak [57], selection of optimal material for solar car by Ghose *et al.* [58]. The procedure of FCOPRAS method includes the following steps:

(i) Construction of HFN decision matrix by the opinion of decision experts. The DM's assigns linguistic terms with respect to the factor.

(ii) The normalized matrix is calculated in the same way as discussed in FTOPSIS method using equation (29).

(iii) Construction of weighted normalized matrix by multiplying the factor weights and the normalized matrix using equation (30).

(iv) Evaluation of the beneficial attributes (BA) and non-beneficial attributes (NBA) denoted as B^+ and B^- respectively as follows:

$$\begin{array}{l}B^+ = \left\{\sum_{p=1}^{m} a_p^{\mathrm{WN}}, \sum_{p=1}^{m} b_p^{\mathrm{WN}}, \sum_{p=1}^{m} c_p^{\mathrm{WN}}, \sum_{p=1}^{m} d_p^{\mathrm{WN}}, \sum_{p=1}^{m} e_p^{\mathrm{WN}}, \sum_{p=1}^{m} f_p^{\mathrm{WN}}\right\} \\ B^- = \left\{\sum_{p=m+1}^{i} a_p^{\mathrm{WN}}, \sum_{p=m+1}^{i} b_p^{\mathrm{WN}}, \sum_{p=m+1}^{i} c_p^{\mathrm{WN}}, \sum_{p=m+1}^{i} d_p^{\mathrm{WN}}, \sum_{p=m+1}^{i} e_p^{\mathrm{WN}}, \sum_{p=m+1}^{i} f_p^{\mathrm{WN}}\right\}\end{array} \tag{34}$$

where equation (30) gives the value of $a_p^{\mathrm{WN}}, b_p^{\mathrm{WN}}, c_p^{\mathrm{WN}}, d_p^{\mathrm{WN}}.e_p^{\mathrm{WN}}, f_p^{\mathrm{WN}}$ and $p = 1,2,\ldots,m$ denotes the beneficial attributes and $p = m+1, m+2, \ldots, i$ denotes the non-beneficial attributes among the alternatives. In this study, only price of the food (F_{11}) is the non-beneficial sub-factor. All others factors and sub-factors are Beneficial attributes in this research. Beneficial criteria are those criteria where enhancement of value will benefit the customers, for non-beneficial criteria it is the reverse.

(v) At the end, defuzzification of HFN is done using the centroid-based method (CBM) of a hexagonal fuzzy number. The value of Sq^{+} for the beneficial attributes and Sq^{-} for the non-beneficial attributes are calculated.

(vi) Finally, calculation the equation:

$$Cq = Sq^{+} + \frac{Sq^{-}_{\min} \times G}{Sq^{-} \times H} \tag{35}$$

where

$$\begin{cases} G = \sum_{q=1}^{j} Sq^{-} \\ H = \sum_{q=1}^{j} \frac{Sq^{-}_{\min}}{Sq^{-}} \end{cases}$$

and $q = 1, 2, \ldots, j$ are the alternatives.

(vii) Now, ranking the alternatives from the above data.

$$R = \frac{Cq}{Cq_{\max}} \times 100\% \tag{36}$$

where Cq denotes the qth defuzzified value and $Cq_{\max}$ denotes the maximum defuzzified value from the considered alternatives.

3.5. Pseudo code depicting the empirical study application

The research model under consideration involving "i" number of alternatives based on "j" number of factor is represented below. The input taken in our study are the preferential linguistic terms assigned by DMs. These variables are converted to HFN for obtaining the output *i.e.*, the ranking of the alternatives. i = Restaurants location as alternative j = Number of factor $i \times j$ = Size of the matrix

Input: The preferential rating matrix in terms of HFN

Output: The ranking order of the restaurants location as alternative in the TOPSIS approach

1. **for** ($p = 1$ to i, $q = 1$ to j) **do**

2. Generate HFN by DMs.

For every given criteria create a matrix and compare the given criteria with each other using linguistic terms in HFN 1–9 scale

3. Calculating criteria weight in HFN by using FAHP.

4. Use HFN AHP methodology to check whether the matrix is consistent or not.

5. If the matrix is consistent, calculate HFN-TOPSIS for ranking of restaurants location as alternatives.

Else, Go back to step 4.

6. Construct normalized values NZ_{ef}

$$\tilde{N} = [n_{pq}]_{ij}, \qquad p = 1, 2, \ldots, i;\ q = 1, 2, \ldots, j;$$

$$N^{B}_{pq} = \left\langle \left(\frac{\tilde{a}_{pq}}{\tilde{f}^{*}}, \frac{\tilde{b}_{pq}}{\tilde{f}^{*}}, \frac{\tilde{c}_{pq}}{\tilde{f}^{*}}, \frac{\tilde{d}_{pq}}{\tilde{f}^{*}}, \frac{\tilde{e}_{pq}}{\tilde{f}^{*}}, \frac{\tilde{f}_{pq}}{\tilde{f}^{*}}; r_{pq}, s_{pq} \right) \right\rangle, \quad d \in \text{B.A.},\ \tilde{f}^{*} = \max f_{pq}$$

$$N^{NB}_{pq} = \left\langle \left(\frac{\tilde{a}^{*}}{\tilde{f}_{pq}}, \frac{\tilde{a}^{*}}{\tilde{e}_{pq}}, \frac{\tilde{a}^{*}}{\tilde{d}_{pq}}, \frac{\tilde{a}^{*}}{\tilde{c}_{pq}}, \frac{\tilde{a}^{*}}{\tilde{b}_{pq}}, \frac{\tilde{a}^{*}}{\tilde{a}_{pq}}; r_{pq}, s_{pq} \right) \right\rangle, \quad d \in \text{N.B.A.},\ \tilde{a}^{*} = \min a_{pq}$$

7. Generate weighted normalized value $P_{pq} = \tilde{N}_{pq} \times \tilde{W}_{q}$;

8. Calculate (FPIS^{+}) and (FNIS^{-})

$$\text{PIS}^{+} = \{a_1^{+}, a_2^{+}, \ldots, a_j^{+}\} = \{(\max a_{pq} | a \in M_B), (\min a_{pq} | q \in M_{NB})\}$$

$$\text{NIS}^{-} = \{a_1^-, a_2^-, \ldots, a_j^-\} = \{(\min\, a_{pq}|a \in M_B), (\max\, a_{pq}|q \in M_{NB})\}$$

9. Calculate distance measure of each alternatives from (PIS^+) and (NIS^-)

$$\tilde{L}_p^+ = \sum_{q=1}^{j} d(P_{pq}, q_p^+),\ \tilde{L}_p^- = \sum_{q=1}^{j} d(P_{pq}, q_p^-)$$

10. Compute relative closeness $R_p = \frac{\tilde{L}_p^-}{\tilde{L}_p^+ + \tilde{L}_p^-}$;
11. **end for**

4. Factor and sub factor for attracting the customers in a restaurant

A restaurant not only attracts people based on the food it has to offer but also due to various other attributes/factors which are enumerated in Figure 5 and discussed below:

4.1. Food (F_1)

To attract customers in a restaurant, the food items in the menu is an important aspect. Great taste of food and consistently maintaining the quality of dishes help in making a strong relationship between customers and the restaurant.

4.1.1. Price (F_{11})

Price sensitive Customers give significance to this attribute. Keeping in mind the preference and needs of this segment of consumers, focusing on price of the food is an important sub- factor.

4.1.2. Food Quality (F_{12})

A restaurant is known by the food it serves. The quality of the food items being offered should match the profile of the target audience to whom it is served. Customer's flock into a restaurant based on various attributes of the food namely (i) Appearance (shape, size, gloss, colour etc), (ii) Oiliness of the food, (iii) Flavour, (iv) Nutritional content, (v) Ethical and sustainable raw materials used etc.

4.1.3. Variety (F_{13})

Variety of food is important as it depends upon the likes and dislikes of the customers. If foods of all variation are available *e.g.* Chinese, continental, north Indian etc. consumers will prefer more to go that specific restaurant which avail all types of cuisine.

4.2. Service (F_2)

In the restaurant industry good service is very important. It may success their overall restaurant business. Good quality service will enhance the frequency of visit by a satisfied customers.

4.2.1. Service Quality (F_{21})

Reliability, responsiveness, tangibility, empathy and assurance are the key dimensions through which service quality is measured. These dimensions of service quality are very important from customer's perspective [59].

4.2.2. Behavior (F_{22})

The first aspect which touches a customer, even before the taste of food hits the tongue is the behaviour of the staff personnel who take down the order as well as deliver the food. If their behaviour is polite then it attracts customers to keep ordering for more food items whereas impolite behaviour is often the ground for low sales of the restaurant.

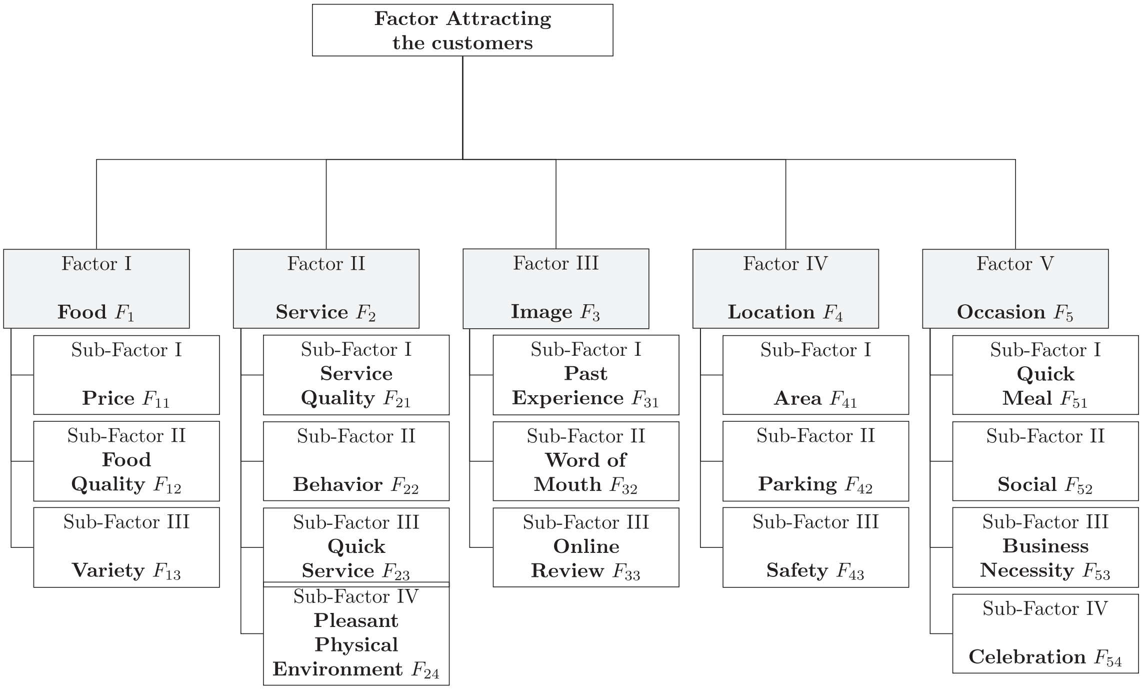

FIGURE 5. *Factor* and *sub-factor* for attracting the customers in a restaurant.

4.2.3. Quick Service (F_{23})

Customers who are in a hurry, mostly want everything in short time. When it comes to restaurants near busy road, railway stations, bus terminus customers prefer places where foods are served quickly and efficiently. The pandemic has impacted the restaurant business worldwide, thus quick service restaurant are the specific ones which serves food within minimal possible time.

4.2.4. Pleasant Physical Environment (F_{24})

Pleasant physical environment symbolises noise, soothing fragrance, music and temperature. Loud and harsh music affects the emotions of customers in restaurants. Soothing light, fragrance enhances customer's mood and emotions and thus influences food consumption. Certain temperature impacts the consumer's behaviour negatively. So, pleasant ambience is a significant factor in restaurant selection and building a long standing relation with the customers.

4.3. Image (F_3)

Image of a restaurant in the eyes of a customers help in building loyalty and new customer base. Strategic marketing, maintaining quality helps in building image.

4.3.1. Past Experience (F_{31})

Past pleasing experience always helps in retaining customers. Satisfactory service leads to a loyal customer base.

4.3.2. Word of Mouth (F_{32})

One will always prefer the restaurant which has earned reputation with time. Customers often opt for the restaurant which has gained a lot positive feedback from their past consumers.

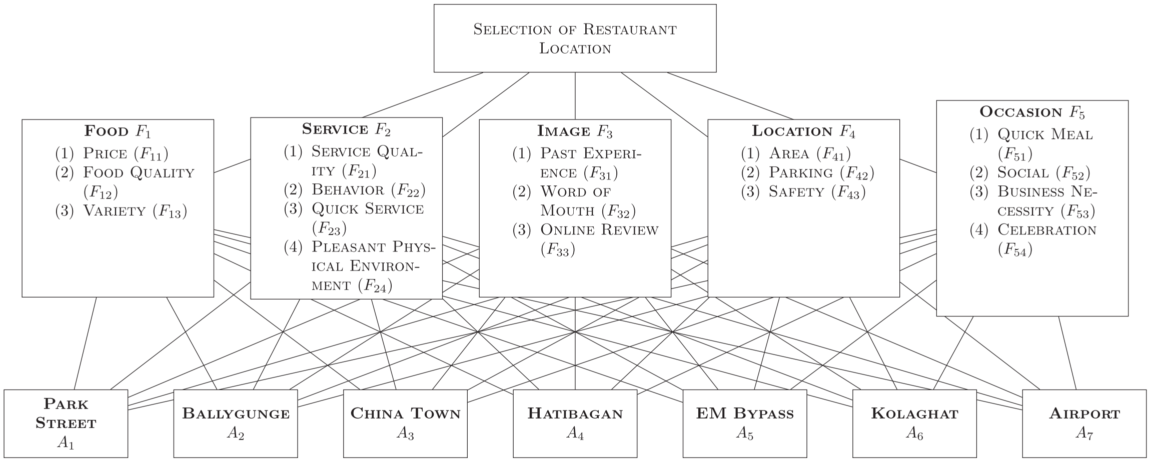

Figure 6. Hierarchical structure representing the factors and sub-factors with alternatives.

4.3.3. Online Review (F_{33})

In this world of technology, people use their smart phones, laptops etc. in order to check the review of the restaurant such as rating, image, menu, direction to have a clear impression on their mind for the particular restaurant. Thus, making it easier and convenient for them to select the restaurant.

4.4. Location (F_4)

The location of the restaurant is important. This location preference varies from one restaurant to the other based on their target audience. In case a restaurant is built without keeping these in mind it will not get a good number of customers.

4.4.1. Area (F_{41})

An adequate area is necessary for a restaurant layout. The consumer seating place should be at an optimal distance and properly ventilated from the restaurant kitchen, pantry and storage room such that the noise of preparation of food and the smell of cooking do not reach the consumers.

4.4.2. Parking (F_{42})

Whenever a customer with own vehicle contemplates about having food in a restaurant the first thought that strikes is about its parking options. A good parking area ensures that the customer will be able to have his meal in peace and feel contended thereby increasing the chances of his coming back to the restaurant again and again.

4.4.3. Safety (F_{43})

The safety factor inside and around a restaurant is important since the customers come to the place to relax and consume food. If there is any kind of compromise on the customer's safety then it won't be long before it goes out of business.

4.5. Occasion (F_5)

Social gatherings, celebrations, quick meals, business events are different occasions for which restaurants are availed to have food service within the venue.

TABLE 3. Alternative consider in this paper and details of those alternatives.

Location	Latitude and longitude	Location	Latitude and longitude
Park Street (A_1)	22.5768 °N; 88.3504 °E	Ballygunge (A_2)	22.5280 °N; 88.3659 °E
China Town (A_3)	22.5739 °N; 88.3556 °E	Hatibagan (A_4)	22.5975 °N; 88.3707 °E
EM Bypass (A_5)	22.4942 °N; 88.4008 °E	Kolaghat (A_6)	22.4352 °N; 87.8607 °E
Airport (A_7)	22.6531 °N; 88.4449 °E		

4.5.1. Quick Meal (F_{51})

For any type of restaurant service for quick meal is a popular strategy for attracting people specially those who are in hurry. The places like train station, airport, busy road adjacent locations, refueling stations etc. are the places where people search for quick meals.

4.5.2. Social (F_{52})

Celebration like ring ceremony, marriage like social occasions requires suitable venue. many a times suitably located restaurant are preferred for such occasions.

4.5.3. Business Necessity (F_{53})

Corporate events and business meets are arranged keeping in mind various commercial dimensions. These events are regularly held in restaurants depending on number of crowds and profile of the attendees. This segment is lucrative part of restaurant business.

4.5.4. Celebration (F_{54})

Birth day, friendship day etc like celebration may be arrange in a restaurant.

Figure 6 represents the hierarchical structure of the numerical study taken into consideration.

5. MODEL SET UP AND CORRESPONDING PROBLEM

Seven location in the state of West Bengal, India is chosen for this study. The locations are: Park Street (A_1), Ballygunge (A_2), China Town (A_3), Hatibagan (A_4), EM Bypass (A_5), Kolaghat (A_6) and Airport (A_7). Their Latitude and Longitude are given in the following Table 5. The satellite location are shown in Figure 7.

Our objective is to rank the locations of restaurant as per preference. We first measure the factors and sub-factors for each location by two decision makers (DM).

5.1. Data source for the study

Data has been collected from customers, restaurant owners and people associated with restaurant business. They were interviewed relating to the questions associated with various important attributes of restaurant and eating pattern. Information regarding factors and sub factors associated with this problem, like the ongoing competitive price associated with the food variety has been collected from two restaurateurs.

5.2. Linguistic terms expressed in HFN in different scale

Linguistic terms are expressed in 1–9 scale and HFN scale for analysing data. Relation between different scale are describe in Table 4.

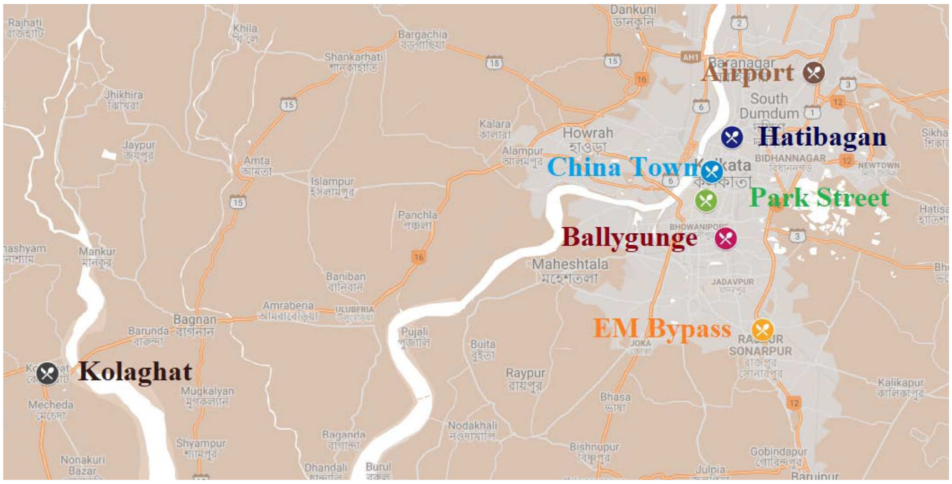

FIGURE 7. Different Restaurant Location around the city of West Bengal, India form Google my map, 2022

TABLE 4. Linguistic terms in hexagonal fuzzy number (HFN) in 1–9 scale.

Linguistic terms	1–9 Scale	Hexagonal fuzzy number (HFN)
Equally important (EI)	1	$(1, 1, 1, 1, 1, 1)$
Moderately important (MI)	3	$(2, 4, 5, 6, 6.5, 7)$
Strongly important (SI)	5	$(3.5, 5, 6, 7, 8, 8.9)$
Very strongly important (VSI)	7	$(6, 7, 7, 8, 9, 9.5)$
Absolutely important (AI)	9	$(7, 8, 9, 9, 9.5, 10)$
Moderately not important (MUI)	1/3	$(1/7, 1/6.5, 1/6, 1/5, 1/4, 1/2)$
Strongly not important (SUI)	1/5	$(1/8.9, 1/8, 1/7, 1/6, 1/5, 1/3.5)$
Very strongly not important (VSUI)	1/7	$(1/9.5, 1/9, 1/8, 1/7, 1/7, 1/6)$
Absolutely not important (AUI)	1/9	$(1/10, 1/9.5, 1/9, 1/9, 1/8, 1/7)$

TABLE 5. Factor matrix for comparison between two design makers (DMs).

Factor	Food (F_1)		Service (F_2)		Image (F_3)		Location (F_4)		Occasion (F_5)	
	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2
Food (F_1)	EI	EI	AI	AI	EI	EI	MUI	MUI	MI	MI
Service (F_2)	AUI	AUI	EI	EI	SI	SI	AI	AI	SI	SI
Image (F_3)	EI	EI	SUI	SUI	EI	EI	MUI	SUI	MI	SI
Location (F_4)	MI	MI	AUI	AUI	MI	SI	EI	EI	MI	MI
Occasion (F_5)	MUI	MUI	SUI	SUI	MUI	SUI	MUI	MUI	EI	EI

Table 6. Sub-factor matrix for Food (F_1)

Sub-factor	Price (F_{11})		Food Quality (F_{12})		Variety (F_{13})	
	DM1	DM2	DM1	DM2	DM1	DM2
Price (F_{11})	EI	EI	EI	EI	MUI	SI
Food Quality (F_{12})	EI	EI	EI	EI	MUI	SI
Variety (F_{13})	MI	SUI	MI	SUI	EI	EI

Table 7. Sub-factor matrix for Service (F_2).

Sub-factor	Service Quality (F_{21})		Behavior (F_{22})		Quick Service (F_{23})		Pleasant Physical Environment (F_{24})	
	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2
Service Quality (F_{21})	EI	EI	SI	SI	EI	EI	SUI	MUI
Behavior (F_{22})	SUI	SUI	EI	EI	VSI	SI	SI	SI
Quick Service (F_{23})	EI	EI	VSUI	SUI	EI	EI	SUI	MUI
Pleasant Physical Environment (F_{24})	SI	MI	SUI	SUI	SI	MI	EI	EI

Table 8. Sub-factor matrix for Image (F_3).

Sub-factor	Past Experience (F_{31})		Word of Mouth (F_{32})		Online Review (F_{33})	
	DM1	DM2	DM1	DM2	DM1	DM2
Past Experience (F_{31})	EI	EI	SI	VSI	SI	VSI
Word of Mouth (F_{32})	SUI	VSUI	EI	EI	SI	SI
Online Review (F_{33})	SUI	VSUI	SUI	SUI	EI	EI

Table 9. Sub-factor matrix for Location (F_4).

Sub-factor	Area (F_{41})		Parking (F_{42})		Safety (F_{44})	
	DM1	DM2	DM1	DM2	DM1	DM2
Area (F_{41})	EI	EI	VSUI	SI	VSUI	VSUI
Parking (F_{42})	VSI	SUI	EI	EI	MUI	VSUI
Safety (F_{44})	VSI	VSI	MI	VSI	EI	EI

5.3. Factor to factor comparison conducted by two DMs

Table 5 describe the opinions of two DMs in linguistic terms. All five factors Food (F_1), Service (F_2), Image (F_3), Location (F_4) and Occasion (F_5) are consider with 1st Decision Maker (DM1) and 2nd Decision Maker (DM2).

Table 6 describes the decision makers review in linguistic terms of the sub-factor Food (F_1). Similarly, Tables 7–10 describes the decision makers review in linguistic terms of the sub-factors Service (F_2), Image (F_3), Location (F_4) and Occasion (F_5) respectively.

TABLE 10. Sub-factor matrix for Occasion. (F_5)

Sub-factor	Quick Meal (F_{51})		Social (F_{52})		Business Necessity (F_{53})		Celebration (F_{54})	
	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2
Quick Meal (F_{51})	EI	EI	SUI	VSI	VSUI	VSI	SUI	EI
Social (F_{52})	SI	VSUI	EI	EI	SUI	VSUI	EI	SUI
Business Necessity (F_{53})	VSI	VSUI	SI	VSI	EI	EI	SI	SI
Celebration (F_{54})	SI	EI	EI	SI	SUI	SUI	EI	EI

TABLE 11. Description of preference of factors in defuzzified form using CBM method.

Factors	Food (F_1)	Service (F_2)	Image (F_3)	Location (F_4)	Occasion (F_5)
Food (F_1)	1.00	8.90	1.00	0.21	5.22
Service (F_2)	0.11	1.00	6.54	8.90	6.33
Image (F_3)	1.00	0.17	1.00	0.20	5.89
Location (F_4)	5.34	0.11	6.01	1.00	5.22
Occasion (F_5)	0.21	0.17	0.20	0.21	1.00
Sum	7.67	10.34	14.74	10.54	23.66

TABLE 12. Representation of the normalized weight of the factors.

Factors	Food (F_1)	Service (F_2)	Image (F_3)	Location (F_4)	Occasion (F_5)
Priority Weight	0.24	0.33	0.11	0.28	0.04

TABLE 13. The priority of factors weight are represented.

Factors	Food (F_1)	Service (F_2)	Image (F_3)	Location (F_4)	Occasion (F_5)	**Sum**
Food (F_1)	0.24	2.89	0.11	0.06	0.22	3.53
Service (F_2)	0.03	0.33	0.73	2.47	0.27	3.82
Image (F_3)	0.24	0.05	0.11	0.06	0.25	0.71
Location (F_4)	1.30	0.04	0.67	0.28	0.22	2.50
Occasion (F_5)	0.05	0.05	0.02	0.06	0.04	0.23

6. NUMERICAL ILLUSTRATION

Table 11 describes the preference of factors in defuzzified form by CBM method using the equation (12). The normalized weight of the factors are represented in Table 12 by using the equation (21).

The weight of the factors are represented in Table 13, using the equation (22). This table shows that the highest weight amongst the factor is Service (F_2), followed by Location (F_4), Food (F_1), Image (F_3) and the least weight is obtained for the factor Occasion (F_5) for selection of the best site for restaurant in Kolkata; the capital city of West Bengal, India.

In Table 14 describe the linguistic variables rating in terms of HFN for alternative rating. Table 15 shows the fuzzy weights of the factors, sub-factors and global weight (sub-factors) in HFN.

Note 2. *In HFN r and s represents the membership function of β_2 and β_5 respectively. Here $r = s = 0.5$ for the numerical application.*

TABLE 14. Linguistic variables represented using HFN for alternative rating with respect to sub-factor.

Linguistic terms	Hexagonal fuzzy number (HFN)
Extremely good (EG)/extremely high (EH)	$(6, 7, 7.5, 8, 8.5, 9)$
Very good (VG)/very high (VH)	$(5.5, 6, 7, 7.5, 8, 8.5)$
Good (G)/high (H)	$(3.5, 4.5, 5, 6.5, 7, 8)$
Poor (P)/low (L)	$(3, 4, 4.5, 5.5, 6.5, 7)$
Very poor (VP)/very low (VL)	$(1, 1.5, 2.5, 3.5, 5, 6)$

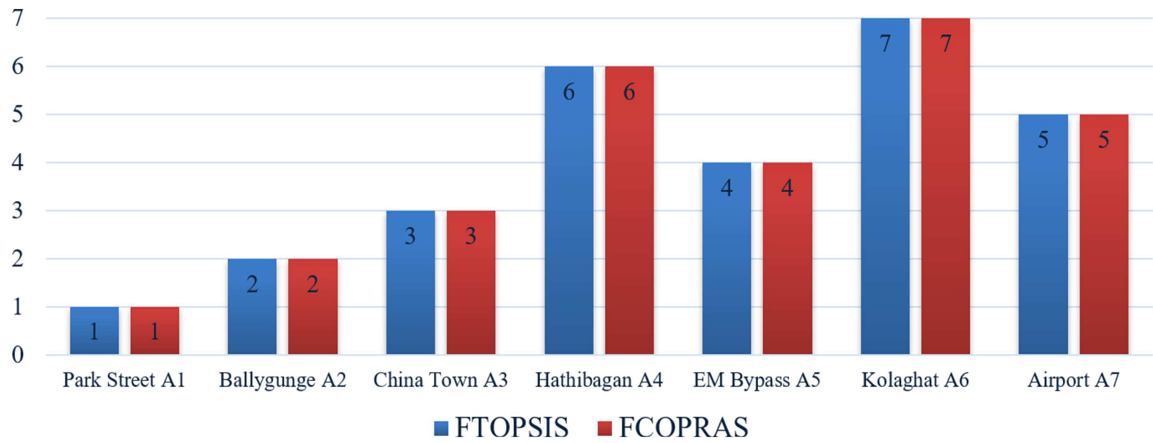

FIGURE 8. Presentation of the ranking obtained by the two MCDM technique FTOPSIS and FCOPRAS.

Tables 16 and 17 describes the comparison table for rating of alternatives in linguistic variables with respect to sub-factors by two decision makers (DMs).

Table 18 describes the positive relative distance and negative relative distance between alternatives and ranking them by evaluating data of relative closeness by FTOPSIS method. Similarly, Table 19 describe the positive relative distance and negative relative distance between alternatives and ranking them by evaluating data of relative closeness by FCOPRAS method.

Remarks 1. Ranking of the best restaurant location depends on several conflicting factors and sub-factors with uncertainty of the selection problem. As, fuzzy logic handle these attributes appropriately, HFN is used in this research to capture the uncertain and imprecise data of the factors, sub-factors and alternatives. HFN is an efficient tool comparative to triangular fuzzy number (TFN), trapezoidal fuzzy number (TrFN) and pentagonal fuzzy number (PFN) which is in the comparative analysis section. The DMs uncertainty can be better captured using HFN. In real life scenario, DMs have to assign linguistic rating for factors and alternatives. These linguistic terms are then transformed to HFN. Finally, HFN is integrated with MCDM tools AHP, TOPSIS and COPRAS to yield weight and ranking of the factors and alternatives respectively. Thus, Considering the problem of this study *i.e.*, ranking of the best restaurant location, we have used HFN.

Figure 8 represented the ranking of alternatives by two MCDM techniques TOPSIS and COPRAS in HFN field.

TABLE 15. Fuzzy weights of factors, sub-factors and global weight (sub-factors) in HFN.

Factor weight	Sub-factor weight	Global weight
$F_1 = (0.14, 0.19, 0.22, 0.26, 0.32, 0.47)$	$F_{11} = (0.07, 0.07, 0.32, 0.37, 2, 2.07)$	$F_{11} = (0.01, 0.01, 0.07, 0.1, 0.63, 0.98)$
	$F_{12} = (0.07, 0.07, 0.32, 0.37, 2, 2.07)$	$F_{12} = (0.01, 0.01, 0.07, 0.1, 0.63, 0.98)$
	$F_{13} = (0.3, 0.03, 0.29, 0.35, 3.48, 3.66)$	$F_{13} = (0.01, 0.01, 0.06, 0.09, 1.1, 1.73)$
$F_2 = (0.18, 0.25, 0.31, 0.35, 0.44, 0.60)$	$F_{21} = (0.12, 0.16, 0.2, 0.25, 0.32, 0.48)$	$F_{21} = (0.02, 0.04, 0.06, 0.09, 0.14, 0.29)$
	$F_{22} = (0.17, 0.24, 0.32, 0.4, 0.52, 0.73)$	$F_{22} = (0.03, 0.06, 0.1, 0.14, 0.23, 0.44)$
	$F_{23} = (0.05, 0.06, 0.08, 0.09, 0.13, 0.2)$	$F_{23} = (0.01, 0.02, 0.02, 0.03, 0.05, 0.12)$
	$F_{24} = (0.13, 0.22, 0.3, 0.38, 0.5, 0.72)$	$F_{24} = (0.02, 0.05, 0.09, 0.13, 0.22, 0.43)$
$F_3 = (0.60, 0.08, 0.10, 0.12, 0.15, 0.24)$	$F_{31} = (0.37, 0.5, 0.68, 0.82, 1.09, 1.38)$	$F_{31} = (0.02, 0.04, 0.07, 0.1, 0.17, 0.34)$
	$F_{32} = (0.11, 0.14, 0.18, 0.22, 0.29, 0.42)$	$F_{32} = (0.01, 0.01, 0.02, 0.03, 0.04, 0.1)$
	$F_{33} = (0.04, 0.04, 0.05, 0.06, 0.09, 0.13)$	$F_{33} = (0.002, 0.003, 0.01, 0.01, 0.01, 0.03)$
$F_4 = (0.11, 0.20, 0.26, 0.31, 0.39, 0.53)$	$F_{41} = (0.03, 0.03, 0.1, 0.12, 0.3, 0.11)$	$F_{41} = (0.004, 0.01, 0.02, 0.04, 0.12, 0.06)$
	$F_{42} = (0.04, 0.04, 0.11, 0.13, 0.37, 0.61)$	$F_{42} = (0.004, 0.01, 0.03, 0.04, 0.14, 0.33)$
	$F_{43} = (0.35, 0.45, 0.7, 0.85, 1.23, 1.64)$	$F_{43} = (0.04, 0.09, 0.18, 0.27, 0.48, 0.87)$
$F_5 = (0.02, 0.03, 0.03, 0.04, 0.06, 0.12)$	$F_{51} = (0.02, 0.01, 0.13, 0.16, 1.31, 0.23)$	$F_{51} = (0.001, 0.001, 0.005, 0.01, 0.08, 0.03)$
	$F_{52} = (0.02, 0.01, 0.08, 0.1, 0.49, 0.62)$	$F_{52} = (0.001, 0.001, 0.003, 0.004, 0.03, 0.07)$
	$F_{53} = (0.12, 0.09, 0.43, 0.52, 2.21, 0.94)$	$F_{53} = (0.003, 0.003, 0.01, 0.02, 0.13, 0.11)$
	$F_{54} = (0.07, 0.04, 0.27, 0.32, 2.08, 2.55)$	$F_{54} = (0.001, 0.001, 0.01, 0.01, 0.12, 0.3)$

TABLE 16. Comparison table in linguistic variables by decision maker 1 (DM1).

	DM1	Sub-factor																
		F_{11}	F_{12}	F_{13}	F_{21}	F_{22}	F_{23}	F_{24}	F_{31}	F_{32}	F_{33}	F_{41}	F_{42}	F_{43}	F_{51}	F_{52}	F_{53}	F_{54}
Alternative	Park Street (A_1)	VH	VG	EH	EH	EH	EH	EG	EG	EG	EG	EG	P	EH	EG	EG	EH	EG
	Ballygunge (A_2)	EH	EG	VH	VH	VG	EH	EG	EG	EG	EG	EG	P	EH	P	EG	H	EG
	China Town (A_3)	H	G	H	H	G	H	G	G	G	G	P	G	H	P	G	VL	G
	Hatibagan (A_4)	L	G	L	L	G	L	P	P	G	P	P	VP	H	G	P	VL	VP
	EM Bypass (A_5)	H	VG	L	L	G	L	G	G	G	G	G	G	H	P	G	H	G
	Kolaghat (A_6)	L	G	L	H	G	L	P	G	P	P	G	G	L	G	P	VL	VG
	Airport (A_7)	L	G	L	H	G	H	G	G	P	G	G	G	L	G	P	VL	G

TABLE 17. Comparison table in linguistic variables by decision maker 2 (DM2).

	DM2	Sub-factor																
		F_{11}	F_{12}	F_{13}	F_{21}	F_{22}	F_{23}	F_{24}	F_{31}	F_{32}	F_{33}	F_{41}	F_{42}	F_{43}	F_{51}	F_{52}	F_{53}	F_{54}
Alternative	Park Street (A_1)	VH	VG	EH	EH	EH	EH	EG	EG	EG	EG	EG	P	EH	EG	EG	EH	EG
	Ballygunge (A_2)	EH	VG	VH	VH	VG	EH	VG	VG	EG	VG	VG	P	EH	P	EG	H	EG
	China Town (A_3)	H	G	H	H	G	H	G	G	G	G	P	G	H	P	G	VL	G
	Hatibagan (A_4)	L	P	L	L	G	L	P	P	G	P	P	VP	H	G	P	VL	VP
	EM Bypass (A_5)	H	G	L	L	G	L	G	G	G	G	G	G	H	P	G	H	G
	Kolaghat (A_6)	VL	G	L	H	G	L	P	G	P	P	G	G	L	G	P	VL	G
	Airport (A_7)	L	G	L	H	G	H	G	G	P	G	G	G	L	G	P	L	VG

TABLE 18. Relative distance between alternatives and ranks by evaluating data of relative closeness by FTOPSIS method.

Alternatives	L_P+	L_P-	$R_p = \frac{L_p^-}{L_p^+ + L_p^-}$	Ranking
Park Street (A_1)	0.038218	0.968535	0.962038	1
Ballygunge (A_2)	0.11621	0.916555	0.887477	2
China Town (A_3)	0.463056	0.571668	0.552483	3
Hatibagan (A_4)	0.684387	0.349598	0.338108	6
EM Bypass (A_5)	0.504143	0.523579	0.509456	4
Kolaghat (A_6)	0.908425	0.12678	0.122468	7
Airport (A_7)	0.584408	0.442658	0.430993	5

TABLE 19. Relative distance between alternatives and ranks by evaluating data of relative closeness by FCOPRAS method.

Alternatives	Sq^+	Sq^-	Cq	$R(\%)$	Ranking
Park Street (A_1)	1.501891	0.038295	1.595873	159.5873	1
Ballygunge (A_2)	1.464773	0.034311	1.569666	156.9666	2
China Town (A_3)	1.252848	0.054506	1.318877	131.8877	3
Hatibagan (A_4)	1.158175	0.062447	1.215808	121.5808	6
EM Bypass (A_5)	1.243346	0.054506	1.309376	130.9376	4
Kolaghat (A_6)	1.189432	0.161916	1.21166	121.166	7
Airport (A_7)	1.207976	0.062447	1.265609	126.5609	5

6.1. Computational complexity

In this section we describe the computational complexity for proposed fuzzy MCDM model. The computational complexity idea is not new (see [60–62]), but here we find the same for our problem. The number of calculation has been use to determine the time complexity which is denoted by T on this problem. We also denoting i as the number of factors, j as the number of sub-factors, k as the number of alternatives and N as the number of decision makers. Therefore the following steps are taking the calculating the computational complexity.

(1) Each FAHP comparison matrix is of i^2 entries, therefore the entries given by N DMs is of $N \times i^2$ entries. To find the comparison matrix need Ni^2 number operations. Then for defuzzification process i^2 operations need and for normalised the defuzzified comparison matrix also i^2 operations performed. Then for nth root and factor weight there are $2i$ operations. Factor weight calculated by i^2 operations also. There after factor

sum and sum/weight calculated by $2i$ operations. Finally consistency ratio calculated by 3 more operations. The total calculation needs $N \times i^2 + i^2 + i^2 + 2i + i^2 + 2i + 3 = (N+3)i^2 + 4i + 3$ number of operations.

(2) For Hexagonal Fuzzy Weight of factors and sub-factors, calculate geometric mean by i^2 operations. For sum, inverse and calculate factor weight we performed $i + i + i^2 = i^2 + 2i$ calculation. Total $i^2 + i^2 + 2i = 2i^2 + 2i$ calculation conducted.

For sub-factor weight, $j = j_1 + j_2 + \cdots + j_i$ number of sub-factors with N decision makers is there. So for the comparison matrix need to calculated $N(j_1^2 + j_2^2 + \cdots + j_i^2)$ operations. For geometric mean conducted $j_1^2 + j_2^2 + \cdots + j_i^2$ operations. Then for sum and inverse there are $2 \times (j_1 + j_2 + \cdots + j_i)$ operation needed. Finally, calculated sub-factor weight by $j_1^2 + j_2^2 + \cdots + j_i^2$ operations. Then total operations of sub-factor weight is $N(j_1^2 + j_2^2 + \cdots + j_i^2) + (j_1^2 + j_2^2 + \cdots + j_i^2) + 2 \times (j_1 + j_2 + \cdots + j_i) + (j_1^2 + j_2^2 + \cdots + j_i^2) = (N+2)(j_1^2 + j_2^2 + \cdots + j_i^2) + 2(j_1 + j_2 + \cdots + j_i)$.

Grand total calculation for factor and sub-factor weight calculation is $2(i^2+i) + (N+2)\left(j_1^2 + j_2^2 + \cdots + j_i^2\right) + 2(j_1 + j_2 + \cdots + j_i)$.

(3) For FTOPSIS method, decision matrix is $k \times j$ entries with N DMs, so there are Njk entries. To construct the decision matrix Njk operations. For findings normalized and weighted normalized decision matrix need to performed $2jk$ operations. For finding positive and negative ideal solution there are $2j$ number of operations conducted. Measure the distance form positive and negative ideal solution there are $2jk$ operations and for calculated total sum $2k$ numbers of operations performed. Finally compassion ratio and ranking the alternatives $2k$ operations needed. Then total $Njk + 2jk + 2j + 2jk + 2k + 2k = (N+4)jk + 2(j+k)$ number of operations performed.

(4) For FCOPRAS methodology, up to weighted normalized decision matrix $Njk + 2kj$ calculation performed. Then calculated sum of beneficial and non-beneficial attributes $2k$ number of operations performed. For defuzzification process $2k$ number of operation conducted. Then calculated Q_i values for k operations conducted. Finally k number of operation performed to rank the alternatives. Total $Njk + 2kj + kj + 2k + 2k + k + k = (N+3)jk + 6k$ number of operations conducted.

Thus the time complexity of this study T is calculated as factor $i = 5$, sub-factor $j = 17$, alternatives $k = 7$ and decision maker $N = 2$ are given as follows

- For FAHP, number of calculations are $(2+3) \times 5^2 + 4 \times 5 + 3 = 148$.
- For weight, number of operations are $2 \times 5^2 + 2 \times 5 = 60$.
- For FTOPSIS, number of operations are $(2+4) \times 17 \times 7 + 2 \times (17+7) = 762$.
- For FCOPRAS, number of calculations are $(2+3) \times 17 \times 7 + 6 \times 7 = 637$.

Then the total time complexity $T = 148 + 60 + 762 + 637 = 1607$.

7. Comparative analysis

Comparative study has been conducted to validate the consistency and robustness of the ranking. To check the efficacy of the methods used, FTOPSIS and FCOPRAS ranking tools are integrated with various uncertain environment *i.e.*, triangular fuzzy number (TFN), trapezoidal fuzzy number (TrFN) and pentagonal fuzzy number (PFN) with respect to hexagonal fuzzy number (HFN).

7.1. Comparative analysis using FTOPSIS method

Table 20 shows the consistency of alternatives ranking using FTOPSIS method inherent with FAHP in the number TFN, TrFN, PFN & HFN and Figure 9 depicts the graphical presentation of the alternatives ranking.

Remarks 2. Form Table 20 and Figure 9 we see that, the FTOPSIS ranking of the alternatives on the four fuzzy numbers are same. Therefore we can conclude that the ranking of alternatives is consistence with respect to HFN also.

TABLE 20. Ranking of the alternatives on the bases of different fuzzy numbers using FTOPSIS method.

Alternatives	TFN	TrFN	PFN	HFN
Park Street (A_1)	1	1	1	**1**
Ballygunge (A_2)	2	2	2	**2**
China Town (A_3)	3	3	3	**3**
Hatibagan (A_4)	6	6	6	**6**
EM Bypass (A_5)	4	4	4	**4**
Kolaghat (A_6)	7	7	7	**7**
Airport (A_7)	5	5	5	**5**

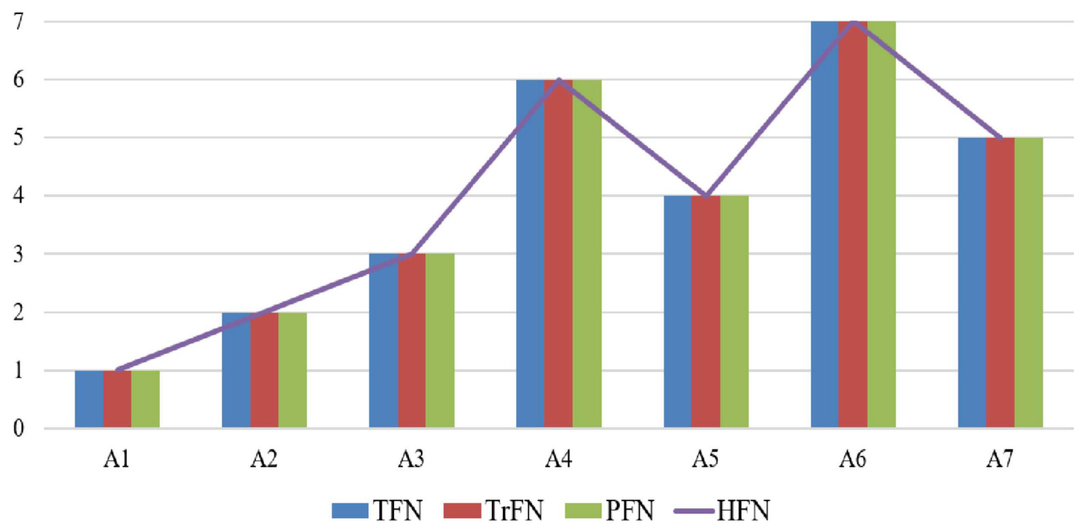

FIGURE 9. Illustration of comparative ranking of the alternatives using TFN, TrFN and PFN with HFN using FTOPSIS.

TABLE 21. The alternatives ranking on the bases of different fuzzy numbers using FCOPRAS method.

Alternatives	TFN	TrFN	PFN	HFN
Park Street (A_1)	1	1	1	**1**
Ballygunge (A_2)	2	2	2	**2**
China Town(A_3)	3	3	3	**3**
Hatibagan (A_4)	7	7	7	**6**
EM Bypass (A_5)	4	4	4	**4**
Kolaghat (A_6)	6	6	6	**7**
Airport (A_7)	5	5	5	**5**

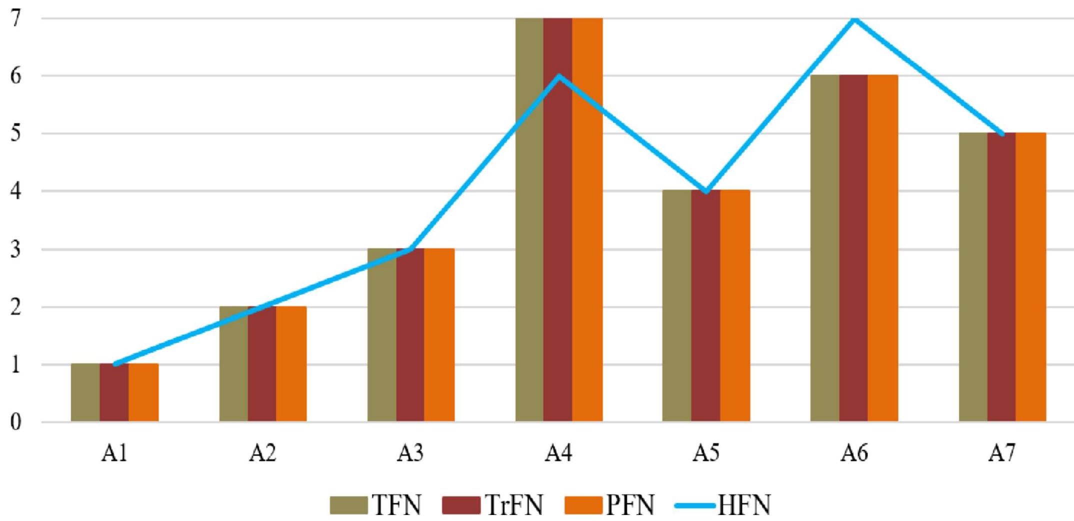

FIGURE 10. Demonstration of alternatives comparative ranking using TFN, TrFN and PFN with HFN using FCOPRAS.

7.2. Comparative analysis using FCOPRAS method

Table 21 exhibit the steadiness of alternatives ranking using FCOPRAS method inherent with FAHP in the number TFN, TrFN, PFN & HFN and Figure 10 depicts the representation of the alternatives ranking diagrammatically.

Remarks 3. Table 21 and Figure 10 shown that the ranking of the alternatives is same for Park Street (A_1), Ballygunge (A_2), China Town (A_3), EM Bypass (A_5) and Airport (A_7) but the position of the alternatives Hatibagan (A_4) and Kolaghat (A_6) is interchanged for fuzzy numbers TFN, TrFN and PFN with HFN respectively. Therefore, from comparative analysis, we conclude that HFN measure uncertainty of the DMs, gives most probable and effective ranking of the alternatives. HFN shows proximity to the optimal result as it is observed that FTOPSIS and FCOPRAS gives the same ranking of the alternatives.

8. Sensitivity analysis

Sensitivity analysis has been carried out to check the sensitivity of rankings based on different priorities for different segment of customers. Our interaction with customers revealed that same customers may behave differently based on the occasion and time availability. Thus the following cases of sensitivity analysis has been taken into consideration.

8.1. Interchange sub-factors Price (F_{11}) and Food Quality (F_{12})

Individual priorities for price of the food and food quality varies. There are customers sensitive to price and some are sensitive to food quality. Thus this interchange of weightage is carried out to conduct the sensitivity analysis.

Table 22 and Figure 11 represents the ranking obtained through the interchange the weights for the two sub-factors "Price (F_{11})" and "Food Quality (F_{12})". It shows the impact of interchanging weights.

TABLE 22. Ranking of two MCDM methods (FTOPSIS and FCOPRAS) by interchange two sub-factors Price (F_{11}) and Food Quality (F_{12}).

Alternatives	FTOPSIS ranking	FCOPRAS ranking
Park Street (A_1)	1	1
Ballygunge (A_2)	3	2
China Town (A_3)	4	4
Hatibagan (A_4)	7	7
EM Bypass (A_5)	6	6
Kolaghat (A_6)	2	3
Airport (A_7)	5	5

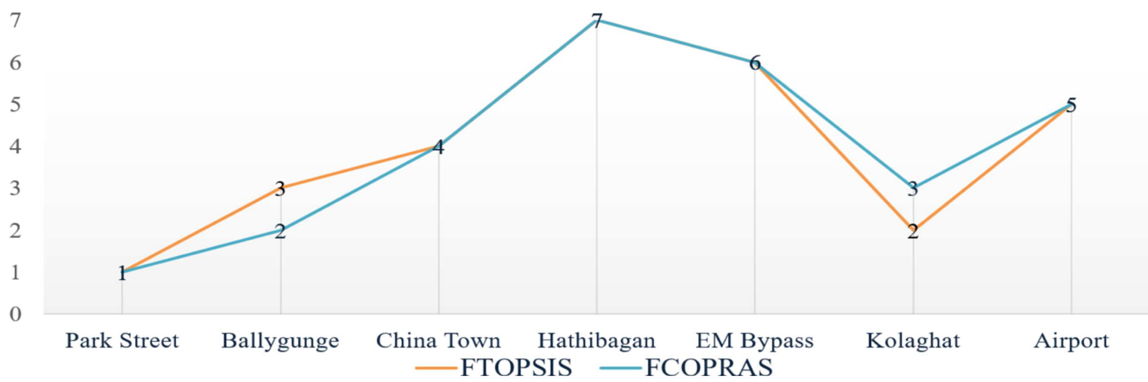

FIGURE 11. *Representation of sensitivity analysis:* Interchange two Sub-factors Price (F_{11}) and Food Quality (F_{12}) the ranking obtained by the two MCDM technique FTOPSIS and FCOPRAS.

8.2. Interchange Service Quality (F_{21}) and Quick Service (F_{23})

For customers with time constraint, quick service will be priority and for customers without time constraint, may behave differently while selecting restaurant. Thus these two weights are interchanged to carry out sensitivity analysis.

8.3. Removing the sub-factor Parking (F_{42})

For a segment of customers, parking facility is not under consideration as they avail public conveyance, hired cars where no parking facility is required. Thus, for such customers parking weight will be 0.

Table 23 represents the ranking obtained through the interchange the weights for the two sub-factors "Service Quality (F_{21})" and "Quick Service (F_{23})" and removing the Sub-factor "Parking (F_{42})". It can be observe removal of Parking has not impacted the ranking significantly.

Figure 12 represent of interchange two Sub-factors "Service Quality (F_{21})" and "Quick Service (F_{23})" and removing the Sub-factor "Parking (F_{42})" effects on Ranking of alternatives by two MCDM methods.

8.4. Removing the sub-factor Price (F_{11})

For a segment of customers price is not under consideration as they have affluence to afford it. Hence food quality is getting more importance and price has no weightage for those customers.

The ranking of Fuzzy TOPSIS and Fuzzy COPRAS are calculated from the hexagonal fuzzy weightage of the sub-factors by removing the sub-factor "Price (F_{11})" are describe in Table 24 and Figure 13.

TABLE 23. Ranking of two MCDM methods (FTOPSIS and FCOPRAS) by interchange two sub-factors Service Quality (F_{21}) and Quick Service (F_{23}) and removing the sub-factor Parking (F_{42}).

Alternatives	FTOPSIS ranking	FCOPRAS ranking
Park Street (A_1)	1	1
Ballygunge (A_2)	2	2
China Town (A_3)	3	3
Hatibagan (A_4)	6	6
EM Bypass (A_5)	4	4
Kolaghat (A_6)	7	7
Airport (A_7)	5	5

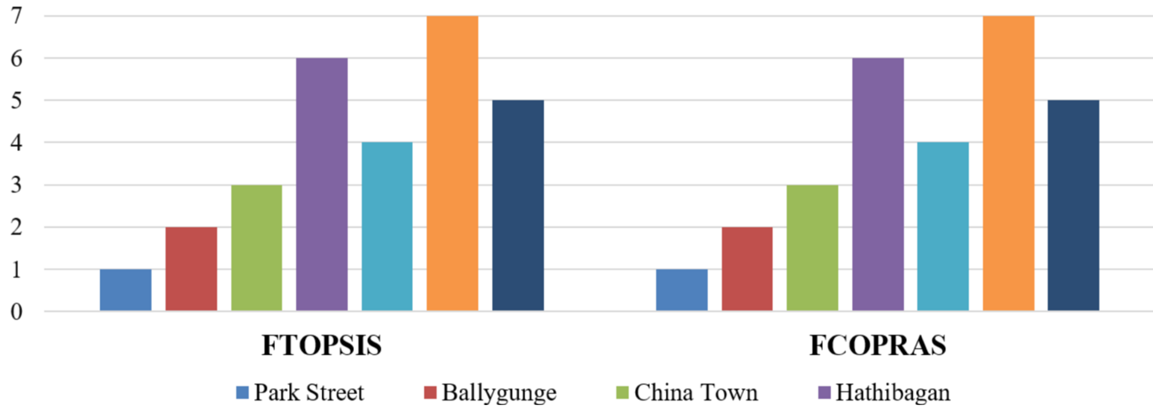

FIGURE 12. *Representation of sensitivity analysis:* Interchange two sub-factors Service Quality (F_{21}) and Quick Service (F_{23}) and removing the sub-factor Parking (F_{42}) give the same ranking obtained by the two MCDM technique FTOPSIS and FCOPRAS.

TABLE 24. Ranking of two MCDM methods (FTOPSIS and FCOPRAS) by removing the sub-factor Price (F_{11}).

Alternatives	FTOPSIS ranking	FCOPRAS ranking
Park Street (A_1)	1	1
Ballygunge (A_2)	2	2
China Town (A_3)	3	3
Hatibagan (A_4)	7	7
EM Bypass (A_5)	4	4
Kolaghat (A_6)	6	6
Airport (A_7)	5	5

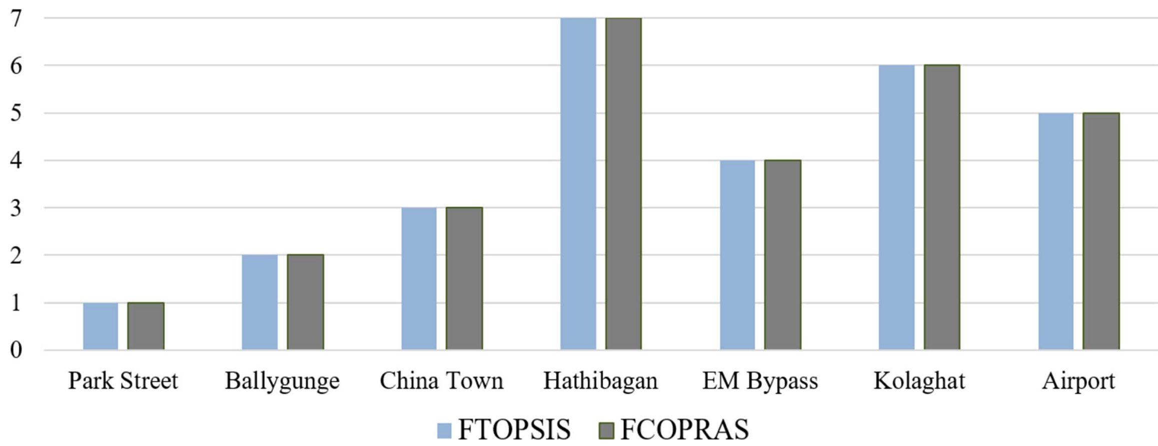

FIGURE 13. *Representation of sensitivity analysis:* Removing the sub-factor Price (F_{11}) the ranking obtained by the two MCDM technique FTOPSIS and FCOPRAS.

TABLE 25. Ranking of two MCDM methods (FTOPSIS and FCOPRAS) by removing the sub-factor Quick Service (F_{23}).

Alternatives	FTOPSIS ranking	FCOPRAS ranking
Park Street (A_1)	1	4
Ballygunge (A_2)	2	1
China Town (A_3)	3	2
Hatibagan (A_4)	6	6
EM Bypass (A_5)	4	3
Kolaghat (A_6)	7	7
Airport (A_7)	5	5

8.5. Removing the sub-factor Quick Service (F_{23})

Many a times, for family outing and other occasions, the customers have plenty of time and hence quick service facility is not under consideration at all.

Table 25 and Figure 14 represents the ranking obtained through the removing the sub-factor "Quick Service (F_{23})".

8.6. Removing the sub-factors Word of Mouth (F_{32}) and Online Review (F_{33})

Customers with prior good experience are not interested in assigning any weightage to word of mouth and online review. Their own excellent experience will be the sole deciding factors for a section of customers.

The rankings using Fuzzy TOPSIS and Fuzzy COPRAS are calculated from the Hexagonal Fuzzy weightage of the sub-factors by removing the Sub-factors "Word of Mouth (F_{32})" and "Online Review (F_{33})" are describe in Table 26 and Figure 15.

8.7. Removing the sub-factor Quick Meal (F_{51})

For restaurant selection, while having plenty of time to spend customers like to enjoy the pleasant environment for longer duration, such situation reduces the quick meal weightage to 0.

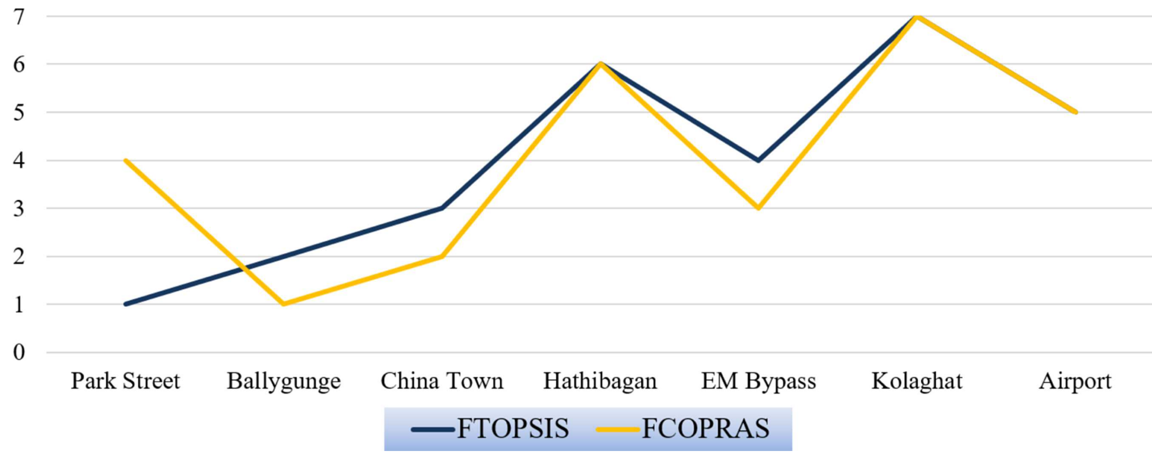

FIGURE 14. *Ranking representation of sensitivity analysis:* Removing the sub-factor Quick Service (F_{23}) the ranking obtained by the two MCDM technique FTOPSIS and FCOPRAS.

TABLE 26. Ranking of two MCDM methods (FTOPSIS and FCOPRAS) by removing the sub-factors Word of Mouth (F_{32}) and Online Review (F_{33}).

Alternatives	FTOPSIS ranking	FCOPRAS ranking
Park Street (A_1)	1	1
Ballygunge (A_2)	2	2
China Town (A_3)	3	3
Hatibagan (A_4)	6	6
EM Bypass (A_5)	4	4
Kolaghat (A_6)	7	7
Airport (A_7)	5	5

8.8. Removing the sub-factor Social (F_{52})

For business and professional outing, for social outing weightage will be 0.

8.9. Removing the sub-factor Business Necessity (F_{53})

For social, family outings, the business necessity weightage will be 0.

8.10. Removing the sub-factor Celebration (F_{54})

For non-celebratory occasion like food for hunger and quick meal, the celebration weightage will be 0.

Above last four case; section 8.7, removing the sub-factor "Quick Meal (F_{51})", Section 8.8, removing the sub-factor "Social (F_{52})", Section 8.9, removing the sub-factor "Business Necessity (F_{53})" and Section 8.10, removing the sub-factor "Celebration (F_{54})" the ranking using Fuzzy TOPSIS and Fuzzy COPRAS are calculated from the hexagonal fuzzy weightage of the sub-factors are give same ranks which are described in Table 26 and Figure 15.

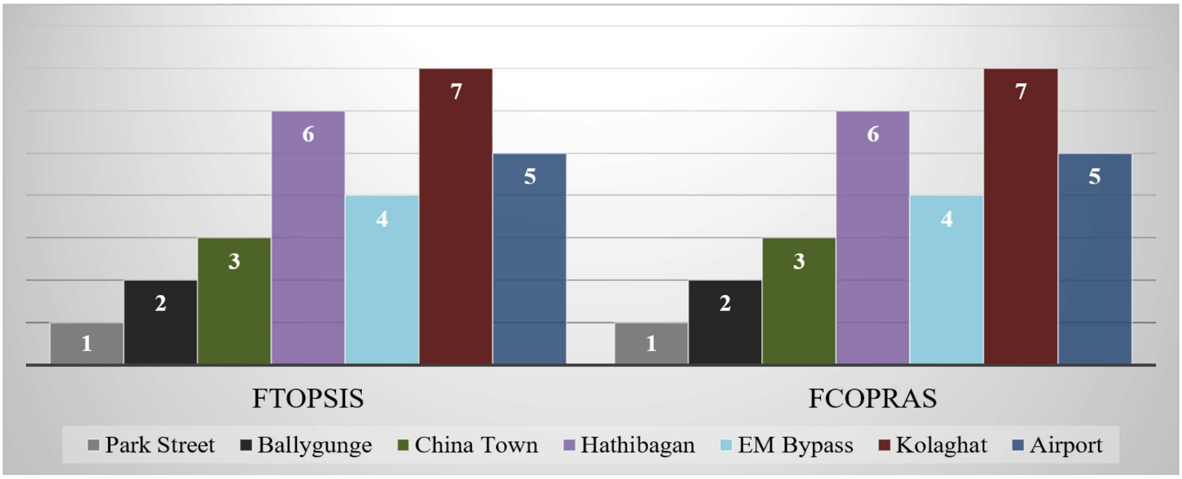

FIGURE 15. *Representation of Sensitivity Analysis:* Removing the sub-factors Word of Mouth (F_{32}) and Online Review (F_{33}) the ranking obtained by the two MCDM technique FTOPSIS and FCOPRAS.

9. Practical implications

The aim of this research is to rank different restaurant locations in the city of Kolkata. For this purpose, we took the data for different factors and sub-factors associated with the said problem by taking hexagonal fuzzy numbers (HFN). This ranking is helpful for entrepreneurs who wish to invest in food vans, food truck, mobile kitchen, mobile canteen, catering truck, food trailer, cloud kitchen and business entities who wish to invest to start restaurant business, location specific utility. It also helps food lover to chose his destination. Similar model can be applied in different cities and problems like tourism location ranking, weekend destination ranking, theme park ranking etc.

10. Conclusion and future research scope

10.1. Summary of problem and contributions

This research aims to evaluate the restaurants location selection using five factors and seventeen sub-factors. The factors and sub-factors play a significant role for selection of restaurants. Seven location of Kolkata, India are chosen as alternatives and those are Park Street, Ballygunge, China Town, Hatibagan, EM Bypass, Kolaghat and Airport. Ranking of location depends on complex and conflicting attributes, experts opinion improves the quality of the decision and help in optimal decision making. HFN is applied to deal with the hesitancy and vagueness of the decision makers (DMs). MCDM tool analytic hierarchy process (AHP) with fuzzy set theory called FAHP is applied to obtain factors and sub factors weight. The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and COmplex PRoportional ASsessment (COPRAS) with Fuzzy environment called FTOPSIS and FCOPRAS respectively are used for ranking of the alternatives. Further, comparative analysis and sensitivity analysis are conducted to examine the sturdiness and robustness of the methods used.

10.2. The major findings

The results obtained through this research are discussed in this section. The ranking obtained under FTOPSIS and FCOPRAS method yields the same ranking of the alternatives. The alternative "Park Street" ranks first position followed by "Ballygunge", "China Town", "EM Bypass", "Airport", "Hatibagan" and "Kolaghat" respectively. Tables 18 and 19 represent the ranking of these methods and Figure 8 graphically demonstrates the ranking. Comparative analysis is performed based on different type of fuzzy number and check consistency and robustness of the alternatives ranking. Table 20 and Figure 9 represent the ranking of alternatives on FTOPSIS method. Similarly, Table 21 and Figure 10 describe the alternatives ranking of FCOPRAS techniques. Sensitivity analysis is conducted, discussed in Section 8 where different cases are considered in which the most sensitive sub-factors weight are interchanged. The results yield through sensitivity analysis reveals the alternative "Park Street" consistently holds the first position. The ranking so attained under sensitivity analysis are represented in Tables 22, 23, 24 and 25. Graphically it is represented in Figures 11, 12, 13 and 14. The findings so obtained using MCDM tools are rational and scientific. It provides future potential work to the researchers.

10.3. The limitations and directions for future research

This research is helpful for entrepreneurs who wish to invest in food vans, food truck, mobile kitchen, mobile canteen, catering truck, food trailer, cloud kitchen and business entities who wish to invest to start restaurant business, location specific utility. One of the limitation of this study is that it involve qualitative assessment on various sub factors like food quality, pleasantness of the environment etc. These qualitative assessments are imprecise and fuzzy in nature. In future research, we can extend the problem with different type of uncertainty settings such as intuitionistic fuzzy, neutrosophic etc. Researchers can explore different sites and find the most important factors and sub factors for a business venture. Similar model can be applied in different cities in other states in India or other countries with different number of alternatives. This study can be applied various problem like tourism location ranking, weekend destination ranking, theme park ranking etc.

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CONTENTS

WILLIAM JAMES' VIEW ON REALITY: A DEFENSE AGAINST THE CHARGE OF INCOHERENCE

MIJANUR RAHAMAN

William James' view on reality: a defense against the charge of incoherence

William James' concept of reality is a unique contribution to the history of metaphysical problem. If we go through the writings of William James we can see that he has discussed about reality from different perspectives and also focused on different aspects of reality. James scholars have often offered various interpretations of James' notion of reality as they tried to defend James' theory against various criticisms. However, some James scholars have not only interpreted James' view of reality but also have shown serious difficulties in James' view. E. K. SUCKIEL is one of them. Suckiel notes that in William James' view reality is of two types -- one is pragmatic reality and the other is metaphysical reality. In her book *The Pragmatic Philosophy of William James*, Suckiel writes that "… he approaches the question of reality from two quite different perspectives; and with each perspective he is addressing a different problem and is concerned with "reality" in a different sense."[1]

In what follows I shall first try to explain James' views about reality in general and then I shall reflect on Suckiel's interpretation on James' view. In Suckiel's interpretation metaphysical reality is pure experience and pragmatic reality is based on it: so metaphysical reality is basic or fundamental. However, in Suckiel's view if we accept the existence of pragmatic reality on the basis of the principle that reality is 'what it is known as', then pure experience will have no place in James' system of metaphysics. Thus she maintains that though these two realities can be shown to be related in certain ways, there still remain serious difficulties regarding their relation. She claims that it is a limitation of James' notion of reality. I will try to show, in this article, why Suckiel's above mentioned claims are unacceptable.

1

William James in his book *The Principles of Psychology* discussed the nature of reality. The second volume of this book includes a chapter entitled 'Perception of reality'. Here the first thing that James says while discussing the notion of reality is that –"The sense that anything we think of is unreal can only come, then, when that thing is contradicted by some other thing of which we think. Any object which remains un-contradicted is ipso facto believed and posited as absolute reality."[2] It means that when we are thinking about an object if that object contradicts any other object that we are thinking of, then the object that has become contradictory turns out to be unreal. Let me elaborate this point with the help of an example. Let us consider a boy who is fantasizing of a winged horse (Pegasus) and tells that the winged horse is in the stable. In this case, in James' opinion, a contradiction will appear in the boy's imagination. James would explain the occurrence of the contradiction in the following way. In case any horse is present in the stable for real, then a relationship between the horse and other things present in the stable will take place. But while fantasizing about the winged horse if we think about its presence in the stable then relationship between the fantasized horse and the other things that remain present in the stable is unlikely to take place. We will hardly find any relationship in that case. If we establish a relation between the concept of fantasized horse and the concept of a real stable and proclaim that the winged horse is present in the stable, then a contradiction will arise between these two instances of thoughts. If anything is present in the real stable then it will be related to other things in the stable. Now if the winged horse is conceived to be present in the stable it should have relation with other things in the stable. But the winged horse, being imaginary, cannot have any relation with other things in the stable. So to think that the winged horse is present in the stable is to admit that the winged horse is both related to other things in the stable and also that the winged horse cannot be related to other things present in the stable. Thus the notion of winged horse leads to contradiction in thought. Accordingly a winged horse will be considered to stand for something unreal because it leads to contradiction. Therefore, the general conclusion for James is that, a fantasized object when thought of in real gives rise to contradiction in thought.

In that case the one which is un-contradictory is considered as real. James claims, "...we cannot continue to think in two contradictory ways at once".[3]

In James' opinion when a contradiction arises between two objects of thoughts then we accept one or the other. The one that is accepted is termed real and the other that is rejected is termed unreal. But, one may ask, if a person's choice determines whether something will be accepted or not then the status of the real turns out to be subjective and if it turns out to be subjective then is there not a probability that a single object can appear to be real to someone and unreal to another person at the same moment of time and, in addition, to a single person at different times? Are not these positions counter-intuitive? What will be James' response to this? It needs to be mentioned here that James' response in this context would depend on his notion of pragmatic reality and on the distinction between pragmatic reality and metaphysical reality as drawn by him. In answering the above question on the basis of the notion of pragmatic reality James claims that reality of an object may depend on a person's choice but at the same time he also notes that making a choice does not depend on the person alone. According to James there are uncountable things in this world and these objects bear certain qualities and it is the presence of these qualities that enable them to emerge as real. Thus sensible objects that can draw our attention are lively, they can create pain and pleasure, can impact our will, can create emotional interest, can occur as causes of the sensations and so on. Through the above contention James intends to mean that the qualities of objects are also responsible for determining the choice and thus also for determining the reality of the chosen object. But James will at once point out that the contradicted thing is not unreal in the sense of being non-existent. James states that the things like this one "...still have existence, though not the same existence, as the real things".[4] To substantiate this discussion

James introduces the notion of "*Universe*" which he also termed "total world". According to James, objects of imagination, objects of illusion, and objects of dreams are all undeniable features of the Universe. James, in this context mentions the existence of seven different worlds. Each of these worlds has its own different objects. These seven worlds together constitute the whole universe. These worlds include the world of sense, The world of science, the world of ideal relations or abstract truths, the world of idols of the tribe, the various supernatural worlds, the various worlds of individual opinion and worlds of 'sheer madness and vagary respectively'[5]. He tells that the objects that we think of can belong to any of these seven worlds. Whenever these worlds appear in the thoughts of different people then the person who is thoughtful regarding a particular world consider it to be real and discards the other worlds from his account for that point of time.

But, one may wonder, which among these seven different worlds and their constituents are real according to James? James's response to the above query, from the viewpoint of pragmatic reality, will be as follows. James, to begin with, will claim that among these seven worlds the one that we attend emerges out as real and the one that we do not attend is not real. He adds that every world has its own reality; however, some of the worlds have more reality compared to others. But he emphasizes that *mere existence* of anything does not assure that it also has a reality. Even if there is existence of something, reality may not be present there as a feature of that thing. If something 'appears' as an object it does not necessarily imply its reality in pragmatic sense. Here, 'to appear as something' means, for James, to be a thing *which is thought of*. It is important to note here that the feature of *being thought of* is a mark of metaphysical reality in James' view. Accordingly merely appearing of an object may exist as a metaphysical reality but that is not practical reality, according to James. But if it is *interesting* or *important* to someone then it may be practical reality too. From this perspective James introduces the notions of two realities– practical reality or pragmatic reality and metaphysical reality. While defining practical reality James says – "...what we need is practical reality, reality for ourselves; and, to have that, an object must not only appear, but it must appear both interesting and important. The worlds whose objects are neither interesting nor important we treat simply negatively, we brand them as unreal."[6] It is notable that the term 'unreal' used in the afore mentioned quotation refers to unreality in pragmatic sense.

For understanding James' notion of the real an important issue that needs to be addressed is whether the features of 'being interesting' and 'being important' stand for necessary conditions or sufficient conditions, or necessary and sufficient conditions for something to be termed real. Here my submission is that the condition of being interesting and the condition of being important are related to Jamesian reality in a complex way. First of all we should note that these two conditions do not always go hand in hand. We can explain it with an example as to how a thing or an object can be interesting without being important or can be important without being interesting. Medicine, for example, which is very important to a patient for staying alive, may not be interesting to her. Similarly a researcher may be interested in a topic which is not related to his research area but not important for his research. Needless to say, however, that certain types of things will satisfy both the condition. If we closely examine James' notion of pragmatic real we will find that merely being interesting will not be a mark of real for a thing unless the thing satisfies some need. So the feature of being interesting is not a sufficient condition for being real. Nor would James consider this feature to be individually necessary too. For James would claim that if a thing satisfies the condition of being important then it would be real even if it is not interesting. But James would at once point out that 'being important' is not a necessary condition for something to count as real because if something is interesting and satisfies our need it will be counted as real even if it is not important. However on this point James will claim that though neither 'being interesting' nor ' being important' is not individually a necessary condition of pragmatic reality, either of them must be present for accepting anything as real. In other words nothing can be pragmatically real if it is neither interesting nor important. James insists that anything to be pragmatically real must possess either the feature of being interesting or that of being important, or both. He relates these features with two aspects of our life namely emotional aspect and active aspect, and claims that the satisfaction of either of these aspects is a necessary condition of pragmatic reality.[7] James would claim that these features of a real thing determine our need of various types. When defining reality James says that, in the relative sense or pragmatic sense reality is that which is connected to our emotional and active life. He maintains that "in this sense, whatever excites and stimulates our interest is real; whenever an object so appeals to us that we turn to it, accept it, fill our mind with it, or practically take account of it, so far it is real for us".[6]

One may, however, observe here that we do many things without emotion, and accordingly the question that will be relevant in those cases is whether James will term the objects of those activities as real or not? In case of pure logic or mathematics emotion rarely exists. So what would be James' position regarding the objects of logic or mathematics? To answer this James states that reality might be admitted in those cases but the degree of reality would be comparatively low. According to James, "…as thinkers with emotional reaction, we give what seems to be a still higher degree of reality to whatever things we select and emphasize and turn to WITH A WILL."[8]. Therefore, the object that we select and emphasize with emotion becomes the higher order reality in pragmatic sense.

The crux of the discussion we have made so far can be put in the following way. James uses the notion of reality in two senses: (a) strict sense or metaphysical sense and (b) relative sense or pragmatic sense. When it is described from the view point of un-contradicted thought it presents real in strict sense and when it is described as something which meets our demands or needs it is termed real in relative or pragmatic sense. It can be observed here that these two definitions given by James can be shown to be mutually complementary. One is the continuation of the other definition. The issues arising from the first definition is found to be clarified in the second definition. To explain, in the first definition it was stated that whatever is contradictory is unreal, and whatever is non-contradictory is real. Here James is considering the term 'contradiction' in logical sense. That indicates, according to James, that if one of the terms of the contradiction is accepted the other has to be rejected because one cannot continue with contradictory thoughts. But if contradiction is found between thoughts of two different objects which one should be accepted as real? Both of them could not be accepted together. The answer to this lies in the other definition. This latter definition says that if something is contradictory then we have an option to choose from the objects involved. Which one to be chosen, will depend on its ability to excite and stimulate our interest. The one that will fulfill our interest will meet our needs and then we will choose that object and that choice will make the things real to us. Thus, the second definition clarifies doubts regarding the first definition.

One may object to James' notion of practical or pragmatic reality as "…whatever excites and stimulates our interest is real."[6] The objection may be that many times objects of illusion also excite and stimulate our interest and in that case if James will consider matters of illusion as real? For a hungry and thirsty person perception of water in a desert can excite and stimulate his interest as a result of which the person may move forward to drink water but may end up in realizing that it was only a mirage. Would James claim the illusory water to be real? The answer to this question is found in James' later work entitled *Pragmatism - a new name for some old ways of thinking*. Here James mentions another characterization of reality. James holds – "Reality is in general what truths have to take account of".[9] The answer to the above objection can be provided from this characterization. If anything even after exciting or stimulating our interest turns out to be false then it cannot be real in James' view. To be real it has to be an object of truth. Thus, even if object of illusion excites and stimulates our interest it will be unreal. However an objection may be put here from, what James calls, the intellectualist view that truth involves the notion of reality and hence there is circularity in James' above mention view of reality. In defense of James it may be argued here that this objection is not well-founded one because James will not accept the correspondence theory of truth *simpliciter*, but will be more inclined to define truth in terms of 'usefulness' or 'workability' of belief and idea.[10] While analyzing James' notion of reality we can observe a continuity in discussion in the two books written by James, namely *The Principles of Psychology-II* and *Pragmatism - a new name for some old ways of thinking*. We can also hold that the discussion in the latter book is the extended version of his previous work. In the book *Pragmatism- a new name for some old ways of thinking*, James talks about various constituents of reality. He says that "By 'realities' or 'objects' here, we mean either things of common sense, sensibly present, or else common-sense relations, such as dates, places, distances, kinds, activities".[11] He further adds that abstract entity and relations of mental ideas are also real. In James' language "Realities mean then either concrete facts or abstract kinds of things and relations perceived intuitively between them. They furthermore and thirdly mean, as things that new ideas of our must no less take account of, the whole body of other truths already in our possession."[12] However, it is important to note here that all these constituents are considered to be pragmatically real only when they satisfy the conditions of interest, importance and that of truth as mentioned earlier, though their *existence* does not depend on these conditions. It is interesting to observe here that James would admit that *the whole* of a pragmatically real object does not depend on the subject's need of any kind. Thus a subject- independent object becomes a constituent of pragmatic reality.

In the book *Some problems of Philosophy: A Beginning of an Introductions to Philosophy* James gives another definition of real.While stating the definition of real according to pragmatic rule James says that "The best definition I know is that which the pragmatist rule gives: anything is real of which we find ourselves obliged to take account in anyway."[13] That means real is something which cannot be ignored. It means that if to understand an experience we cannot but admit certain constituents then all those constituents would enjoy the status of real object. For example, in James' opinion, for explaining an experience we need to admit both percept and concept, and thus both percept and concept are to be admitted as real. This definition reveals quite clearly that pragmatic reality is not merely a subjective phenomenon but it is regulated by extra-subjective factors or certain factors in subject-object relationship which explains the compulsion of admitting certain objects or parts of objects as real. James' account of metaphysical reality has been elaborated in *Essays in Radical Empiricism.* Here James describes reality in the following way. He says: "there is only one primal stuff or material in the world, a stuff of which everything is composed, and if we call that stuff 'pure experience,' then knowing can easily be explained."[14] Explaining th1e notion of 'pure experience' James says

"'Pure experience' is the name which I gave to the immediate flux of life which furnishes the material to our later reflection with its conceptual categories… which is not yet any definite what, tho' ready to be all sorts of whats; full both of oneness and of manyness …".[15]

According to James, pure experience is neither mental nor physical. It may be called 'mental' if viewed from one perspective and 'physical' from another perspective, it depends on our perspective. Different types of dualism like mind-body, subject-object, and knower-known are only distinguishable functions within pure experience. Pure experience is a stream of experience which James considers to be blind by itself; it cannot say anything about itself. We speak for them. In James' opinion pure experience may function as a *thought* in some context while it can function as *thing* in another context. For example, when paint is within a pot in a paint shop, it may be counted as a thing that is to be sold. On the other hand when it is applied on a canvas along with other paints it becomes an element having, aesthetic appeal. In a similar way the same piece of experience plays the role as thought in one context and in another context plays the role as thing. James gives an excellent illustration of this. He says, "A sensation is rather like a client who has given his case to a lawyer and then has passively to listen in the courtroom to whatever account of his affairs, pleasant or unpleasant, the lawyer finds it most expedient to give."[16]

Though it is evident from Jamesian notion of metaphysical reality that it makes ample room for the admission of pragmatic reality which can be brought out from pure experience once the existence and importance of the subject of experience is admitted as a part of pure experience. James scholar, however, point out that it is very difficult to offer a coherent view of James' notion of reality if we take into consideration the various characterisation of reality found in James' writings. Suckiel, to mention one, draws our attention to some such difficulty and she claims that the relationship between pragmatic reality and metaphysical reality cannot be comprehended if we give equal weightage to all the characterisation of the reality mentioned by James. In the next section I shall deal with Suckiel's understanding of James notion of reality to see which aspects of James' account appear problematic to her.

2

Ellen Kappy Suckiel in the chapter entitled "James's conception of reality" of her book *The Pragmatic Philosophy of William James* undertakes a critical study on James' conception of reality. She observes that the diverse claims made by James regarding the nature of reality have led to various types of disagreement and uncertainty among the James scholars. According to Suckiel, James' view on the nature of reality is the most complex and tangled part of his philosophy. She emphasizes that James has discussed the notion of reality from two different perspectives. According to her, "…James's view comes from the fact that he approaches the question of reality from two quite different perspectives; and with each perspective he is addressing a different problem and is concerned with "reality" in a different sense."[17] As already mentioned these two senses are respectively the metaphysical sense and the pragmatic sense. On the basis of these two senses James distinguishes between pragmatic reality and metaphysical reality.

James' reality in pragmatic sense is bipolar, according to Suckiel. She says that if we go through James' presentation of the notion of pragmatic real we find that on one hand there is teleological subject which has interest and values while on the other hand there is flux of experience. An individual selects from a chaotic stream of experience and arranges them as per her interest and then constructs a comprehensive worldview. She reminds us that for James pragmatic reality to a person is that which brings satisfaction in his life. On this point Suckiel emphasizes that this constructed reality is a non-basic type of reality compared to the status of metaphysical reality.

In metaphysical sense, as claimed by Suckiel, pure experience is the sole and ultimate reality which underlies common sense world. Suckiel adds that, "James calls his metaphysical theory "radical empiricism".[18] Describing radical empiricism Suckiel says that this type of empiricism is characterized by two major claims namely, "everything real must be experienceable", and "every kind of being experienced must somewhere be real". So whatever is real must be experiential as well. Suckiel claims that on James' view *ultimate* metaphysical reality is "pure experience". This experience is continuous, flowing, and an ever-expanding reality. Using different types of concepts we analyze the flux of experience to meet our daily needs. While analyzing stream of experience we cut it off them with concepts. But conceptual thoughts can never present pure experience correctly. In Suckiel's opinion these conceptual thoughts "…stand only for distinguishable functions within experience itself." [19]

After having examined the two senses of reality as admitted by James, Suckiel observes that James has left unaddressed the relationship between these two senses of reality. As a James scholar Suckiel wants to fill up this gap in James' account. Accordingly she proposes her view regarding the relation between pragmatic reality and metaphysical reality. On the basis of her reading of James she claims that pragmatically real objects are constructed by the subjects from the flux of experience. She also contends that in James' scheme pure experience is the only material stuff of the world and pragmatic reality is created from that stuff by the subject. If there is no stuff other than pure experience then pragmatic reality is not possible. On the basis of this understanding of James' view of metaphysical reality Suckiel claims that pure experience is ontologically basic and pragmatic reality is formed by acting upon it. In this sense pragmatic reality depends on pure experience or metaphysical reality. In this case their relation is compatible. Suckiel however points out that in some places James has characterized reality as what it is known as and she argues that if pragmatic reality be characterized in this way then pure experience can no more be regarded as its real stuff because pure experience cannot be considered as 'known as'. Suckiel arrives at this conclusion on the basis of James' claim that pure experience is undifferentiated and is free from subject-object distinction within it. Suckiel would insist that if something cannot be considered to be an object then, it cannot be characterized as 'known as'. Accordingly Suckiel concludes that no satisfactory account of relation between pragmatic real and metaphysical real can be given if we try to develop a coherent account of James' theory of reality by taking into consideration *all* the characterization of the real mentioned by

James. My submission is that Suckiel's interpretation of James is based on certain misunderstanding of what James intended to say regarding reality.

In what follows I shall try to show that the notion of pragmatic reality is integrally related to the notion of pure experience itself and pure experience is also pragmatically meaningful.

3

From the definition of pragmatic reality as given by William James in his various books we can identify certain criteria or conditions of pragmatic reality. If some object, be it physical or mental, fulfils these conditions, then these objects can be termed pragmatically real.

Among those criteria, the first criterion is that an object (concrete or abstract) has to be independently real. James mentions that the first part of reality is the flux of our sensations. We do not have any control on the matters of these perceptions. We can perceive when the stream of this perception arrives, but its arrival is not in our control. When the perceptual flux influences us then we do not have any control over their nature and quantity. We cannot change the sequence and quantity according to our will. Accordingly the objects which are experienced as a part of this flux must have some existence independent of the subject.

The second criterion is that something can be termed pragmatic real only when it is able to satisfy our interest and need. According to James this need may be emotional, active, intellectual and aesthetic. Even though the components of reality are fixed, we have the right to deal with them. The perceptions are undoubtedly out of our control. But the part of perception which will be attended, noted down and emphasized upon will depend on our interest, and reality will emerge depending on the part that received attention.

Thirdly, to be real it has to be an object of truth, or it must be consistent with or at least related to previous truth. However, if it is neither an object of truth nor related to previous truth then it is not real. We have already mentioned that if an object or fact satisfies our need but does not satisfy the requirement of truth then it cannot be pragmatically real. Since the world is changing continuously we have new kinds of experience. Contradiction or inconsistency may arise between previous truth and new experience and then we may require revising the previous truth if the new experience satisfies other criteria of pragmatic reality and if it can coherently relate to the larger body of truth already possessed by the subject.

Suckiel is right in her view that pure experience is ontologically prior to pragmatic reality and the basis of it. But we would argue that though pragmatic reality is based on pure experience the latter cannot be claimed to be the sole basis of the former. If any object is termed pragmatically real then it is a function of pure experience. But pure experience cannot turn out to be a substance or function by itself unless the subject of experience exercises its function of attention, selection etc. the items of pragmatic reality cannot turn into existence. So, pure experience as such cannot be the sole element responsible for the production of pragmatic reality. As a matter of fact the possibility of pragmatic reality is contained in pure experience as such. But the actualisation of pragmatic reality involves certain process which can take place only when the *purity* of pure experience surrenders itself to the divisive activity of the subject. We would claim that this interpretation of the relation between metaphysical reality and pragmatic reality is much more consistent with the spirit of Jamesian philosophy than the interpretation offered by Suckiel. Suckiel's reading of James suggests that James will be prepared to accept the reduction of pragmatic reality to pure experience. But throughout his philosophy James has opposed both dualism and reductionism. So to emphasize *only* on the metaphysical real or pure experience as being something of metaphysical import is to go against James' spirit.

On this point one may wonder that even though the existence of metaphysical reality is significant for the existence of pragmatic reality, can pragmatic reality be in any sense significant for the existence of metaphysical reality as pure experience? Our contention here is that the Jamesian notion of metaphysical reality contains two presentations of pure experience: pure experience as *the possibility of the pragmatic real* and pure experience as *the basis of actualised pragmatic reality.* If pure experience is taken in the form of first presentation then it is undifferentiated though containing the possibility of differentiation. In the second presentation, however, pure experience is to be viewed as flow of experience in which the various elements are differentiated where some differentiated element takes the role of a subject and the others are its object. Since this differentiated state is synonymous with pragmatic reality, it can be claimed that pragmatic reality is significant for the second form of presentation of pure experience itself. It is in this sense that James' notion of metaphysical reality has integral connection with pragmatic reality. From one viewpoint metaphysical reality is significant for pragmatic reality while from another viewpoint pragmatic reality is significant for metaphysical reality. This in its turn brings out the metaphysical significance of pragmatic reality in James' philosophy.

We will conclude this article by pointing out where suckiel goes wrong in her observation regarding the limitation of James' view which she thinks to be due to James' description of reality as something 'known as'. In our opinion Suckiel has taken the expression 'known as' in the sense of propositional knowledge which is descriptive in nature. But if we go through the various writings of James [20] we will find that James was not very happy with the expression 'known as' because of its association with propositional knowledge only. Specifically in the context of radical empiricism when James uses the expression 'known as' he means 'whatever is experienced'. In the editorial preface to James' *Essays in Radical Empiricism* R.B. Perry has clearly mentioned this point to bring out how the principle of reality as 'known as' when taken in the sense of reality as experienced relate pragmatism to radical empiricism. Since pure experience is all through a state of experience James will consider pure experience as something experienceable and it is in this sense he would also consider pure experience to be something 'known as' though not in the sense of propositional knowledge.

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Foot-notes

1 Suckiel, Ellen Kappy (1942), p. 122
2 James, William (1891), pp. 288-89
3 James, William (1891), pp. 288-89
4 Ibid, p. 291
5 Ibid, p. 293
6 Ibid, p. 295
7 James emphasizes on the disjunctive nature of the two features, viz 'being intereting' and 'being important' has been implicated in, Ibid, p. 295
6a James, William (1891), p. 295
8 Ibid, p. 297
9 James, William (1921), p. 244
10 For James' explication of the notion of truth which is relevant for the present context, see p. xi, xiv, 60
11 James, William (1921), p. 206
12 James, William (1921), p. 212
13 James, William (1916), p. 101
14 James, William (1912), p. 4
15 Ibid, pp. 93-94
16 James, William (1921), p. 246
17 Suckiel, Ellen Kappy (1982), p. 122
18 Suckiel, Ellen Kappy (1982), p. 132
19 Ibid, p. 134
20 James, William (1909)
James, William (1912)
James, William (1921)

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Comparative Analysis of Digital Reference Services from Academic Library Websites

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Abstract

Digital reference service is one of the interactive methods of communication between librarians and users. It is an internet-based library reference or question-answer service that connects users with skilled subject experts or reference librarians. This research is an attempt to study the present scenario of digital reference services provided by different academic libraries and to check the status of Indian academic libraries. The objectives of the study are to test the availability of digital reference services on different academic library websites and make a comparative study. In this research 50 Indian and 50 international academic libraries are selected and a comparative analysis is made using six digital reference service forms including Chat Reference, E-mail Reference, Web Form, Text Reference, Phone Reference and Frequently Asked Questions. The study reveals that the top global academic libraries evaluate digital reference services with a strong positive attitude, better to say, define it as indispensable at the same time, the Indian top academic libraries stand the opposite.

Keywords: Academic Libraries, Digital Reference Service, Virtual Reference Service

1. Introduction

The world's libraries and information centres have seen a rapid expansion of new technologies in recent years, particularly in the areas of information technology, communication technology, and large storage technology. The amount of information available and its accessibility have substantially risen because of these technological developments. People must be able to access information resources round-the-clock and from any location. Using user credentials, the library grants access to information in a variety of ways. But what should be done if users require assistance when searching for and retrieving information? The reality is that many users and consumers lack technological sophistication. A significant portion of users who access library online resources are unaware of how to download, sort necessary papers, or conduct searches. We are all aware that reference service entails a personal, human-mediated exchange of information between a user and a reference librarian. As early as 1876 Samuel Swett Green defined Reference service as "Personal Relation between Librarians and Readers". However, how will human connection take place if customers access library content online? A digital or virtual reference service was launched to address the issue of customers who use library resources but are not physically present in the library. According to Lankes (2004), a digital reference is a network of expertise, intermediaries, and resources made available to a user seeking information online. The terms "virtual reference," "digital reference," "e-reference," "Internet information services," "live reference," and "real-time reference" are used interchangeably to describe reference services that make use of computer technology in some way, according to the IFLA Digital Reference Guidelines. (International Federation of Library Associations and Institutions, 2005).

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With the development of the telephone, asynchronous modes of communication for virtual reference were established, but they are no longer the only options. The reference librarian can link users in a synchronous mode of communication thanks to the abundance of web 2.0 technologies that are currently available.

2. Literature Review

A digital reference service, in contrast to the conventional library reference service, enables users to submit inquiries and receive responses over the Internet and other electronic means of communication. Users can connect with librarians or other information specialists, such as topic matter experts, and get the immediate help they need whenever they require it. In digital reference, there are typically four components: users, librarians and information specialists, the interface (such as a website, chat programme, online form, etc.), and information resources. Basics and overview of digital reference services, including what it is, why they exist, and how they are discussed by different researchers in various works of literature (Gray, 2000; Stahl, 2001; Katz, 2002). The terms "digital reference," "virtual reference," and "online reference" all relate to the delivery of library reference services via electronic channels, including asynchronous methods like e-mail and web forms and real-time ones like chat, web push, co-browsing, voice over IP, etc. (Jin *et al.* (2005)). Khan (2006) explores the ideas and problems associated with using digital references in academic libraries, including its history, usage patterns, effects on users and libraries, and possibilities for the future. In their study, Wang and Yuan (2011) discuss the demand for interactive reference services in medical college and university libraries as well as teaching hospitals. Virtual reference services are examined by Nicol and Crook (2013) as a crucial requirement for an academic library. According to Einasto's (2014) study, there should be some fundamental standards for offering digital reference services, including usability, access dependability, security, speed, legitimacy, applicability of e-information, clarity, and responsiveness. In their study comparing in-person and online transactions, Carlo and Yoo (2007) found that in-person transactions involved substantially more positive techniques from librarians.

In their essay, Kadir *et al.* (2006) analyzed several models and kinds of digital reference services. The capacity of the user to submit questions and receive answers electronically is the key feature of digital reference service, which may be provided through real-time chat or asynchronous email. They discussed the many services offered by academic libraries, including links to other OPACs, dedicated websites, online catalogues, FAQ pages, interactive services, and remote access to resources. Another article conducts research on web-based reference services in academic libraries by looking at advertisements for reference services and other digital reference services on the websites of various institutions (Yang & Dalal, 2015; Uutoni, 2018). A case study of virtual reference services in academic libraries spanning IIMs and IITs in India was conducted by Maharana and Panda (2005). In their study, Baro *et al.* (2014) looked into the many avenues that reference librarians in university libraries in Nigeria used to respond to reference requests from users. de Groote *et al.* (2005) researched to ascertain the viability of combining virtually different topic specialities and reference departments within a big university library to produce a single digital reference service. A survey was conducted by Malik and Mahmood (2013) to determine the infrastructure required for digital reference services at university libraries in Punjab, Pakistan. A survey of 150 academic library websites was also done by Janes *et al.* (1999) to determine the number of libraries offering digital reference services and their characteristics. The piece also looked at policies, technological impediments, FAQs, and other online resources. Through a national survey conducted via academic library home pages, Bao (2003) investigates the state of web-based interactive reference services in the United States.

Hervieux (2021) investigates the impact of the COVID-19 pandemic on the questions received through a chat reference service at a university library in Canada. Garvey (2021) makes a comparative study of reference practices before the covid 19 pandemic and during the campus closure and tries to show how consortia' digital reference service has significantly increased during the pandemic situation.

Vladoiu *et al.* (2023) in their study try to examine if there is evidence of gender bias while providing e-mail reference services in public and academic libraries in the USA. Hamer (2021) also investigates the effects of any kind of implicit racial bias or discrimination displayed in virtual reference interactions among academic libraries in England. Adetayo (2023) tries to investigate the potential of AI chatbots in academic libraries. The researcher reveals that ChatGPT can aid with user services like answering basic reference questions. Islam *et al.* (2021) in experimental research examine the 'customization process of Zoho chat in the ABCD site module' to provide a virtual reference service from the library webpage. Tsang and Chiu (2022) in their study explore the users' perceptions and preferences of digital reference services in an academic library in Hong Kong through a qualitative study. Mawhinney (2020) examines user preferences regarding digital reference services using NVivo qualitative data analysis software. An exploratory study was done by Agee (2019) using Language Style Matching (LSM), a text analysis technique to evaluate 1200 email threads to determine the level of engagement between library personnel and users. After a thorough review of the literature, it was discovered that there has been no research comparing the best academic libraries in India with those in other parts of the world, which is why the current study was conducted.

3. Objectives and Research Questions

3.1 The Objectives of the Study

- To test the availability of digital reference services on selected academic library websites.
- To identify the service status by examining various forms, service hours, etc.
- To make a comparative study of digital reference services among selected Indian and global academic libraries.

3.2 Research Questions

What forms are generally used by the global and Indian academic libraries to provide digital reference service? How do academic libraries provide 24*7 digital reference services to their users? What are the basic differences between the Indian academic libraries and the academic libraries of other countries regarding digital reference?

4. Limitations in Scope

For the investigation, a total of 100 academic library websites were used. 50 of the 100 websites are from India, and the remaining 50 are from other countries. The top fifty websites from the National Institutional Ranking Framework (NIRF) 2021, published by the Ministry of Human Resource Development, Government of India, are considered when choosing an academic library in India. The top fifty academic library websites worldwide as listed in Times Higher Education's 2021 World University Rankings are then followed. There is no additional rating system used. Sites were visited from April through May of 2022.

5. Methodology

In this study, the availability of digital reference services on academic library websites is compared and analyzed. Both ordinary university libraries and technical university libraries are considered. The following six types of digital reference services, identified from the research literature and library websites are taken. These forms include the Chat Reference, Email Reference, Web Form for Ask-A-Librarian, SMS or Text Reference, Phone Reference, and FAQ or Knowledge Base. The central library homepages of the institutes are considered the chief sources of information. First, all the numerous Digital Reference Forms that are accessible on the websites of various academic institutions' libraries were thoroughly examined. Using various parameters, all services with detailed information are listed. After analyzing the forms, a comparative evaluation is conducted. No further verification is offered in this regard. Only the information offered on websites is considered as the source of information.

6. Data Collection, Analysis and Findings

The research identifies various types of digital reference services, each with their nature, working hours, user-specific services, textual presentation of the services provided, placement of the service text on library websites, alternative methods of service communication, and other characteristics.

This graph (Figure 1) was created using data from the institution's website and positively assessed in areas with at least one type of digital reference service. According to the graph, 98 per cent of the top academic libraries worldwide and 24 per cent of the top Indian academic libraries offer some sort of digital reference service. Based on the information in Figure 1's data, additional research was conducted.

All the information in Figure 2 was acquired in line with the instructions on academic library websites.

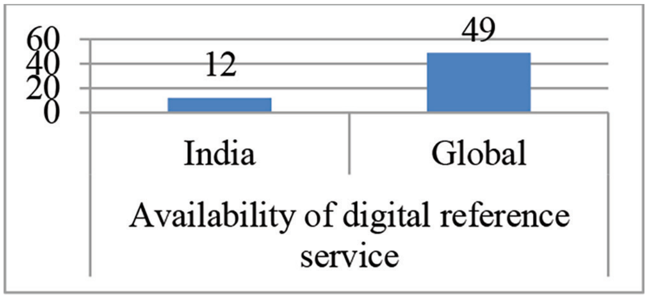

Figure 1. Availability of digital reference services in academic library websites (n-50).

No more testing is carried out to verify that the services are accessible. According to the data, most good academic libraries around the world offer the Ask-A-Librarian (Web Form), Email and Phone Reference services. Only 2% of Indian academic libraries and 72% of all academic libraries globally provide online (real-time) Chat Reference service. However, in the context of digital reference, only 20% of libraries globally provide research support as a special focus. The general state of digital reference services in India is poor.

7. Service Hours for Digital Reference

Figure 3 shows that just 20% of Indian libraries provide digital reference services, and even then, only during work hours (which is usually less than 8 hours). Every workday, 60% of libraries around the world provide digital reference services for 8 hours. On weekdays, 12% of providers operate for fewer than 8 hours, while 6% operate for more than 8. According to the statistics, 13% of libraries provide online reference services for more than 8 hours each week, as well as on Sundays and other holidays. Only 10% of libraries worldwide and 80% of Indian libraries disclose information.

8. Textual Presentation of Digital Reference Link

The terms of a given service are presented in text in their conventional and well-known forms. For the benefit of consumers, the overarching term "Digital Reference Service" or "Virtual Reference Service" encompasses

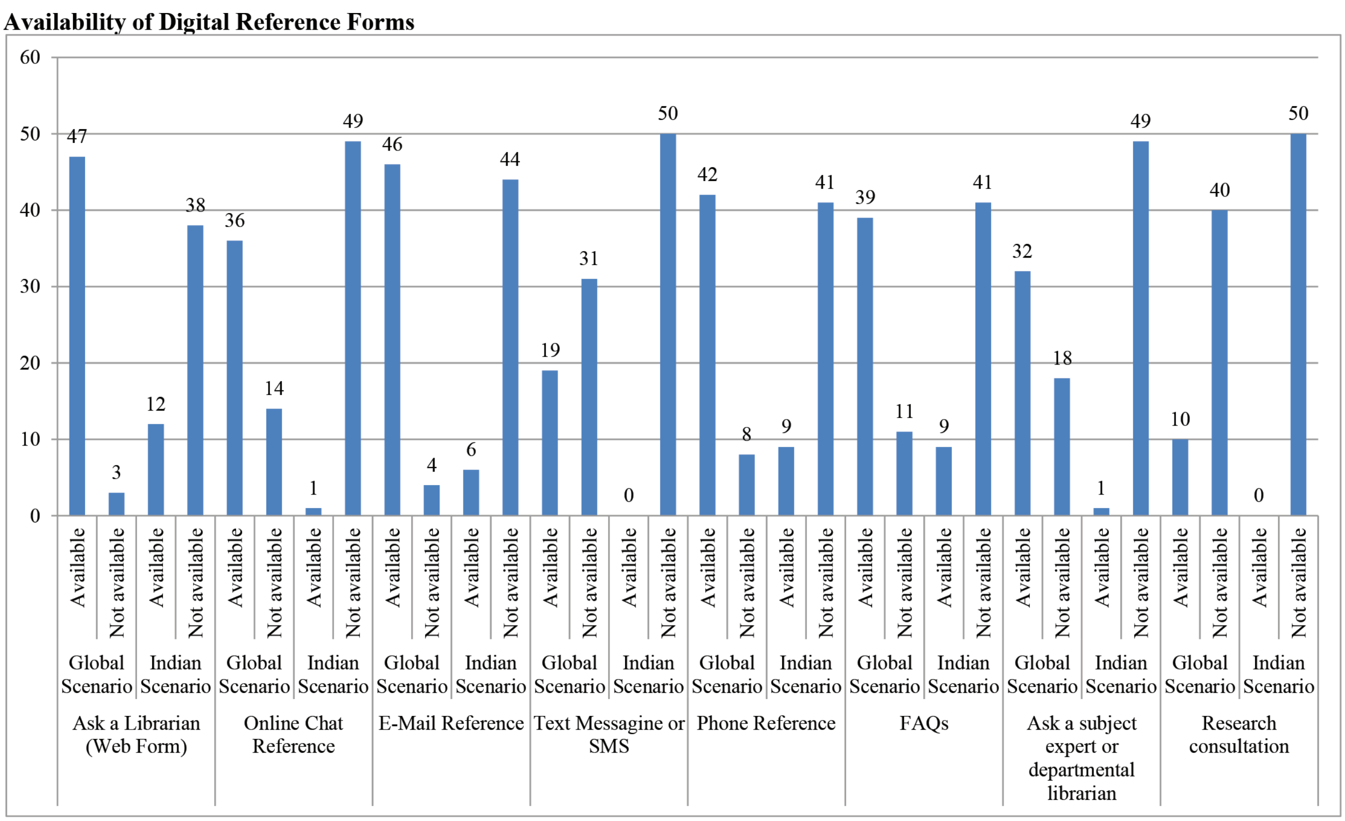

Figure 2. Availability of digital reference forms in the library websites (n=50).

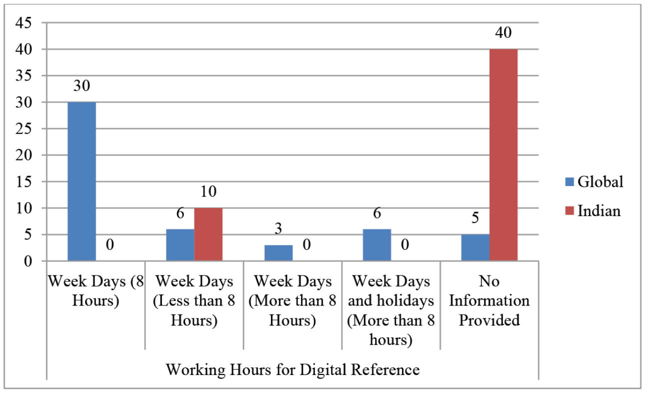

Figure 3. Service hours for digital reference services.

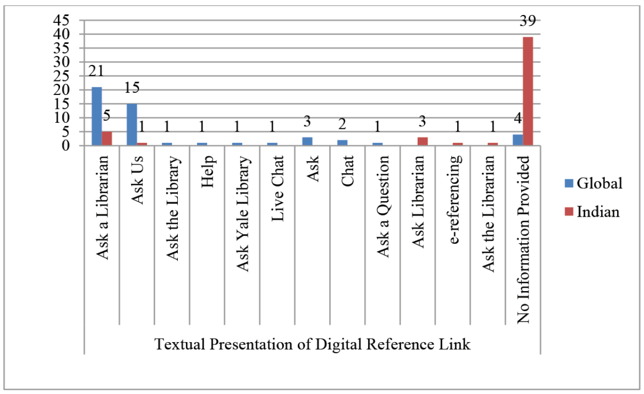

Figure 4. Textual presentation of digital reference service link (n=50).

several well-known terms. According to Figure 4, 10% of Indian libraries and 42% of all libraries worldwide use the text "Ask a Librarian". The percentage of libraries that use "Ask Us" is 30% worldwide and 2% in India. Only 2% of libraries use textual presentations for digital reference services such as "Ask the Library," "Live Chat," "Ask Question," "Help," "Ask the Librarian," "e-referencing," and so on. Six per cent of libraries globally utilize "Ask", whereas six per cent of libraries in India use "Ask Librarian." Instead of utilizing a broad service name, some libraries use specialized service names such as "Chat".

9. Location of Digital Reference Service Link on Library Homepage

Users are often discouraged and exit the page after making a minor effort if they cannot find the relevant information on the website. Mu *et al.* (2011) provided the parameters for the investigation. Users are usually

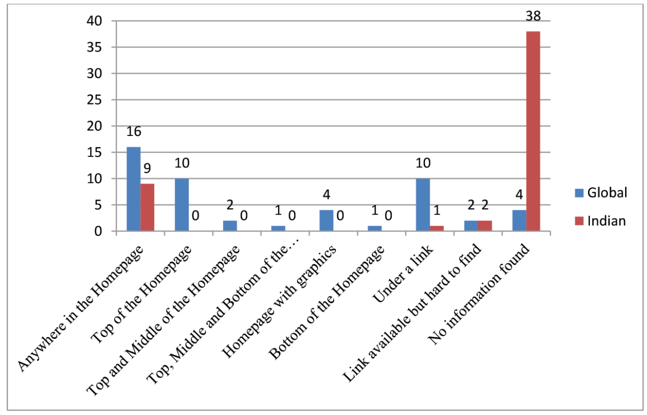

Figure 5. Location of digital reference service link on the library homepage (n=50).

lured to homepages that offer a digital reference link with a graphical image quickly. If the link is at the top of the homepage, access is made easy. According to the study, 20% of library homepages have a link to a digital reference resource at the top of the web page, and these are all from academic libraries outside India. There are no libraries in India that have the DRS link prominently displayed on their web pages. In 4% of all libraries globally, a DRS link may be found on the site in two places: at the top and in the middle. On its webpage, one library provides DRS access to three locations. According to the study's findings, the majority of global libraries that offer DRS prefer to place the link to the service at the top and on the homepage. When the information or service in question is shown at the top of the homepage, it is the most valuable component of the website.

10. Alternative Contact when Live Help is Offline

When live service is unavailable, the study's findings, illustrated in Figure 6, give additional pathways for users to contact reference librarians. According to the statistics, just 8% of libraries in India have alternative communication channels, compared to 80% of libraries worldwide. Among these are email, web forms, text or SMS messaging, and various alternative methods of interaction.

11. Discussions and Findings

The study demonstrates that India's academic library system is still poor in terms of digital reference services. The best institutions in India do not give DRS the attention

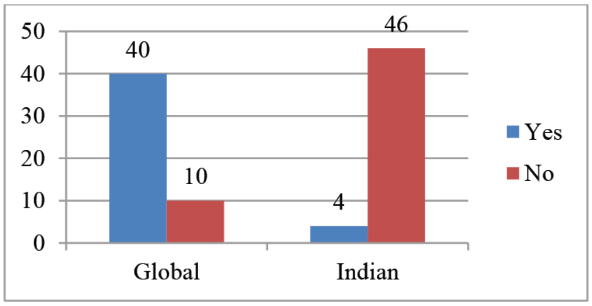

Figure 6. Alternative contact methods during chat offline.

it deserves, whereas the best universities elsewhere do. This result demonstrates that the websites of the main Indian universities' libraries correspond to the "bad" category. According to digital reference testing, most academic libraries worldwide prefer to offer digital reference services via Ask-a-Librarian, email, and phone. The second highest priority is the online chat reference service. However, only a few academic libraries in India provide an "Ask a Librarian" or FAQ service (through a web form), and many libraries do not provide any kind of digital reference service. When compared to other academic libraries, Indian libraries only use one or two modalities of digital reference, such as Ask-a-Librarian, Chat Reference, Email Reference, Phone Reference, Text Messaging, and FAQs. Most libraries throughout the world prefer to use email because it is a straight forward means of communicating in the modern era. Because it is also linked to email, the Ask-a-Librarian Web Form is regarded as the second most favoured service form. Reference librarians typically respond to emails within 24 hours.

Most global libraries produced a thorough list of commonly asked questions and their solutions, which were then uploaded online as a searchable knowledge base. The knowledge base, which is a personalized list of questions and answers that is available 24 hours a day, has a high user satisfaction percentage. To avoid duplication of effort, a reference librarian may seek to respond to requests submitted by email, chat, or other modes of communication. Many university libraries around the world offer subject-specific reference services to their patrons. Users can use this feature to direct questions to a departmental librarian or subject-matter specialist rather than a reference librarian. Users can get the greatest answers from specialists by using this way of communication. Some prestigious institutions engaged a variety of professors and researchers as subject reference specialists to get the best answers.

The study also showed that most libraries worldwide provide DRS for more than eight hours each day. Outside of usual library hours, reference librarians occasionally assist patrons. Libraries in cooperative networks periodically provide digital reference services around the clock, including on holidays. When a digital reference service is provided for fewer hours than the legal minimum, the supplier believes the service to be less important.

The usage of common terminology benefits the promotion of a service. According to Figure 4, the most regularly used terms for DRS are "Ask a Librarian" and "Ask Us." Considering the service, these terms are enticing and straightforward to understand. The majority of global libraries prefer the word "ask," which is deemed the most appropriate in this situation. "Ask Us," "Ask a Librarian," "Ask the Librarian," "Ask a Question," and so on all begin with "Ask." According to the survey, rather than employing dictionary words, foreign libraries prefer to use phrases that are already familiar with and establish them as standard language. If most libraries use the same or a similar phrase for the same type of service, it will be easy to grasp.

The location of any essential link on a webpage, like terminology, is critical. The user will find it more enticing and time-saving if they can immediately reach the link they require. Several of the world's most distinguished libraries, according to the research, prominently display a link to their digital reference service on their webpage. This suggests that the link is easily accessible to the user. Some libraries display many links, which appear at random on the webpage. This service demonstrates that these libraries see their digital reference service as the most important component. To focus visitors' attention on the link, a visual image promoting the service is sometimes provided alongside it. In some library websites, the digital reference link is located in a submenu. Although many libraries provide various types of digital reference aid, most customers use the service when doing an OPAC search. Only 10% of all libraries globally provide links for OPAC searches and no such libraries in India. The link may appear on the discovery service's home page or near the search box on occasion.

12. Conclusion

The primary goal of the research is to determine the availability of various digital reference services on various academic library websites. According to the study, the Indian academic library community belongs to a very weak, unstable platform, as opposed to worldwide academic libraries, and most global libraries provide digital reference services more successfully utilizing an average of six types. The most important synchronous reference service that connects reference librarians with users in real time is chat reference. E-mail is often regarded as the most practical and user-friendly asynchronous way of digital reference service. Without a doubt, service hours are an essential concern, and most libraries are dealing with this time-consuming issue despite having sufficient staff and matching technology. Again, some libraries are attempting to avoid this problem by utilizing a collaborative network. Textual presentation or terminology, as well as its location on the library's webpage, are also important issues because they help users identify and use the DRS link directly and fast. It is expected that this evaluation will help to understand the present situation of digital reference services throughout the world and will draw the attention of different libraries for self-assessment and self-enrichment to cope with the rushing queries of the users smoothly which, undoubtedly, will bring about a strategic change.

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